CFHR 3-5-72-154-3F

THE STRENGTH OF ANCHOR BARS: A REEVALUATION OF TEST DATA ON DEVELOPMENT LENGTH AND SPLICES

By

C. O. Orangun, J. O. Jirsa, and J. E. Breen

RESEARCH REPORT 154-3F

PROJECT 3-5-72-154

COOPERATIVE HIGHWAY RESEARCH PROGRAM
WITH TEXAS HIGHWAY DEPARTMENT
AND
U. S. DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION

CENTER FOR HIGHWAY RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN
JANUARY 1975

RESEARCH REPORTS PUBLISHED BY THE CENTER FOR HIGHWAY RESEARCH

52-1F-"Design Criteria for Overhanging Ends of Bent Caps — Bond and Shear," by Phil M. Ferguson, August 1964.
53-1F-"Development Length for Anchor Bolts," by John E. Breen, April 1964.
54-1F-"A Portable Electronic Scale for Weighing Vehicles in Motion," by Clyde E. Lee and Nasser I. Al-Rashid, April 1968.
55-1 -"Relationship Between Critical Mechanical Properties and Age for Structural Lightweight Concrete," by William B. Ledbetter and J. Neils Thompson, February 1964.
52-2 -"Volume Changes in Unrestrained Structural Lightweight Concrete," by James T. Houston and J. Neils Thompson, August 1964.
55-3F-"Critical Mechanical Properties of Structural Lightweight Concrete and the Effects of These Properties on the Design of the Pavement Structure," by William B. Ledbetter, Ervin S. Perry, James T. Houston, and J. Neils Thompson, January 1965.
56-1 -"A Finite-Element Method of Solution for Linearly Elastic Beam-Columns," by Hudson Matlock and T. Allan Haliburton, September 1966.
56-2 -"A Computer Program to Analyze Bending of Bent Caps," by Hudson Matlock and Wayne B. Ingram, October 1966.
56-3 -"A Finite-Element Method of Solution for Structural Frames," by Hudson Matlock and Berry Ray Grubbs, May 1967.
56-4 -"A Computer Program to Analyze Beam-Columns Under Movable Loads," by Hudson Matlock and Thomas P. Taylor, June 1968.
56-5 -"A Finite-Element Method for Bending Analysis of Layered Structural Systems," by Wayne B. Ingram and Hudson Matlock, June 1967.
56-6 -"Discontinuous Orthotropic Plates and Pavement Slabs," by W. Ronald Hudson and Hudson Matlock, May 1966.
56-7 -"A Finite-Element Analysis of Structural Frames," by T. Allan Haliburton and Hudson Matlock, July 1967.
56-8 -"A Finite-Element Method for Transverse Vibrations of Beams and Plates," by Harold Salani and Hudson Matlock, June 1967.
56-9 -"A Direct Computer Solution for Plates and Pavement Slabs," by C. Fred Stelzer, Jr., and W. Ronald Hudson, October 1967.
56-10-"A Finite-Element Method of Analysis for Composite Beams," by Thomas P. Taylor and Hudson Matlock, January 1968.
56-11-"A Discrete-Element Solution of Plates and Pavement Slabs Using a Variable-Increment-Length Model," by Charles M. Pearre, III, and W. Ronald Hudson, April 1969.
56-12-"A Discrete-Element Method of Analysis for Combined Bending and Shear Deformation of a Beam," by David F. Tankersley and William P. Dawkins, December 1969.
56-13-"A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs," by John J. Panak and Hudson Matlock, January 1970.
56-14-"A Direct Computer Solution for Plane Frames," by William P. Dawkins and John R. Ruser, Jr., May 1969.
56-15-"Experimental Verification of Discrete-Element Solutions for Plates and Pavement Slabs," by Sohan L. Agarwal and W. Ronald Hudson, April 1970.
56-16-"Experimental Evaluation of Subgrade Modulus and Its Application in Model Slab Studies," by Qaiser S. Siddiqi and W. Ronald Hudson, April 1970.
56-17-"Dynamic Analysis of Discrete-Element Plates on Nonlinear Foundations," by Allen E. Kelly and Hudson Matlock, January 1970.
56-18-"A Discrete-Element Analysis for Anisotropic Skew Plates and Grids," by Mahendrakumar R. Vora and Hudson Matlock, August 1970.
56-19-"An Algebraic Equation Solution Process Formulated in Anticipation of Banded Linear Equations," by Frank L. Endres and Hudson Matlock, January 1971.
56-20-"Finite-Element Method of Analysis for Plane Curved Girders," by William P. Dawkins, June 1971.
56-21-"Linearly Elastic Analysis of Plane Frames Subjected to Complex Loading Conditions," by Clifford O. Hays and Hudson Matlock, June 1971.
56-22-"Analysis of Bending Stiffness Variation at Cracks in Continuous Pavements," by Adnan Abou-Ayyash and W. Ronald Hudson, April 1972.
56-23-"A Nonlinear Analysis of Statically Loaded Plane Frames Using a Discrete Element Model," by Clifford O. Hays and Hudson Matlock, May 1972.
56-24-"A Discrete-Element Method for Transverse Vibrations of Beam-Columns Resting on Linearly Elastic or Inelastic Supports," by Jack Hsio-Chieh Chan and Hudson Matlock, June 1972.
56-25-"A Discrete-Element Method of Analysis for Orthogonal Slab and Grid Bridge Floor Systems," by John J. Panak and Hudson Matlock, May 1972.
73-1 -"High-Speed Road Profile Equipment Evaluation," by W. Ronald Hudson, January 1966.
73-2 -"A Profile Measuring, Recording, and Processing System" by Roger S. Walker, Freddy L. Roberts, and W. Ronald Hudson, April 1970.
73-3 -"Pavement Serviceability Equations Using the Surface Dynamics Profilometer," by Freddy L. Roberts and W. Ronald Hudson, April 1970.
73-4 -"Analog-to-Digital System," by Roger S. Walker and W. Ronald Hudson, April 1970.
73-5F-"Development of a System for High-Speed Measurement of Pavement Roughness, Final Report," by Roger S. Walker, W. Ronald Hudson, and Freddy L. Roberts, May 1971.
76-1F-"Fatigue Strength of 3/4 Inch Studs in Lightweight Concrete (Push-out Tests)," by H. G. Lehman, H. S. Lew, and A. A. Toprac, May 1965.
77-1 -"Fatigue Testing of Ribbed Orthotropic Plate Bridge Elements," by H. L. Davis and A. A. Toprac, May 1965.
77-2F-"Fatigue Tests of Welded Hybrid Plate Girders Under Constant Moment," by H. S. Lew and A. A. Toprac, January 1967.
78-1F-"Evaluation of Traffic Control at Highway Intersections," by Clyde E. Lee and Walter C. Vodrazka, March 1970.
79-1F-"Numerical Methods for Radial Triangulation," by Robert D. Turpin, May 1966.
80-1F-"Strength Effect of Cutting Off Tension Bars in Concrete Beams," by Phil M. Ferguson and Syed I. Husain, June 1967.
88-1F-"Factors Affecting Anchor Bolt Development," by D. W. Lee and J. E. Breen, August 1966.
89-1 -"Field Testing of Drilled Shafts to Develop Design Methods," by Lymon C. Reese and W. Ronald Hudson, April 1968.
89-2 -"Instrumentation for Measurements of Lateral Earth Pressure in Drilled Shafts," by Lymon C. Reese, James Crozier Brown, and Harold H. Dalrymple, September 1968.
89-3 -"Studies of Shearing Resistance Between Cement Mortar and Soil," by John W. Chuang and Lymon C. Reese, May 1969.
89-4 -"The Nuclear Method of Soil-Moisture Determination at Depth," by Clarence J. Ehlers, Lymon C. Reese, and James N. Anagnos, June 1969.
89-5 -"Load Distribution for a Drilled Shaft in Clay Shale," by Vasant N. Vijayvergiya, W. Ronald Hudson, and Lymon C. Reese, March 1969.
89-6 -"Instrumentation for Measurement of Axial Load in Drilled Shafts," by Walter R. Barker and Lymon C. Reese, November 1969.
89-7 -"The Determination of Soil Properties *in Situ*," by David B. Campbell and W. Ronald Hudson, November 1969.
89-8 -"Behavior of Axially Loaded Drilled Shafts in Beaumont Clay," by Michael W. O'Neill and Lymon C. Reese, December 1970.

(Continued inside back cover)

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle "The Strength of Anchor Bars: A Reevaluation of Test Data on Development Length and Splices"		5. Report Date January 1975
		6. Performing Organization Code
7. Author(s) C. O. Orangun, J. O. Jirsa, and J. E. Breen		8. Performing Organization Report No. Research Report 154-3F
9. Performing Organization Name and Address Center for Highway Research The University of Texas at Austin Austin, Texas 78712		10. Work Unit No.
		11. Contract or Grant No. Research Study 3-5-72-154
12. Sponsoring Agency Name and Address Texas Highway Department Planning & Research Division P. O. Box 5051 Austin, Texas 78763		13. Type of Report and Period Covered Final
		14. Sponsoring Agency Code

15. Supplementary Notes

Work done in cooperation with the Federal Highway Administration, Department of Transportation.
Research Study Title: "Factors Affecting Splice Development Length"

16. Abstract

An equation has been developed for calculating lengths of lap splices of deformed bars from a nonlinear regression analysis of test results of beams with lap splices. It reflects the effect of length, cover, spacing, bar diameter, concrete strength, transverse reinforcement, and moment gradient on the strength of lap splices. The equation is also applicable in determining basic development lengths. Based on the equation developed, design recommendations are proposed for development lengths and lap splices and compared with AASHTO Interim Specifications for Bridges, 1974. The comparison shows that for the most unfavorable splice conditions (a clear cover of 1-1/2 in. on sides or bottom, splices with no transverse reinforcement, all bars spliced in a region of maximum moment, and bar spacing less than 6 in. on centers) AASHTO provisions overestimate lap lengths by 11 percent for #6, 16 percent for #8, and 25 percent for #11 bars. If cover is increased to 3 in. or transverse reinforcement is added, the splice length of large bars may be reduced by as much as 60 percent over that required by present AASHTO provisions. Furthermore, the equations governing development length are essentially the same as those for splice length.

17. Key Words lap splices, deformed bars, test, beams	18. Distribution Statement

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	90	

Form DOT F 1700.7 (8-69)

THE STRENGTH OF ANCHORED BARS: A REEVALUATION OF TEST DATA ON DEVELOPMENT LENGTH AND SPLICES

by

C. O. Orangun
J. O. Jirsa
J. E. Breen

Research Report No. 154-3F

Research Project 3-5-72-154
Factors Affecting Splice Development Length

Conducted for

The Texas Highway Department

In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by

CENTER FOR HIGHWAY RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

January 1975

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

P R E F A C E

This report presents an extensive study of data on lap splices and development lengths with the aim of developing design provisions for inclusion in AASHTO Specifications.

This is the final report on work conducted under Project 3-5-72-154, "Factors Affecting Splice Development Length." Reports 154-1 and 154-2 describe experimental work conducted under this program. The program was sponsored by the Texas Highway Department and Federal Highway Administration, and administered by the Center for Highway Research at The University of Texas at Austin. Close liaison with Texas Highway Department has been maintained through Mr. Wesley Pair and with the Federal Highway Administration through Mr. Jerry Bowman.

This study, made while the principal author was on sabbatical leave from the University of Lagos, Nigeria, was under the general direction of Professor J. E. Breen and the immediate supervision of Professor James O. Jirsa. Special thanks are due to Professor Breen for giving the principal author an opportunity to participate in the program and also for his continued interest and advice. There were extensive discussions during this study with Professor Phil M. Ferguson, whose suggestions are gratefully acknowledged.

A B S T R A C T

An equation has been developed for calculating lengths of lap splices of deformed bars from a nonlinear regression analysis of test results of beams with lap splices. It reflects the effect of length, cover, spacing, bar diameter, concrete strength, transverse reinforcement, and moment gradient on the strength of lap splices. The equation is also applicable in determining basic development lengths. Based on the equation developed, design recommendations are proposed for development lengths and lap splices and compared with AASHTO Interim Specifications for Bridges, 1974. The comparison shows that for the most unfavorable splice conditions (a clear cover of 1-1/2 in. on sides or bottom, splices with no transverse reinforcement, all bars spliced in a region of maximum moment, and bar spacing less than 6 in. on centers) AASHTO provisions overestimate lap lengths by 11 percent for #6, 16 percent for #8, and 25 percent for #11 bars. If cover is increased to 3 in. or transverse reinforcement is added, the splice length of large bars may be reduced by as much as 60 percent over that required by present AASHTO provisions. Furthermore, the equations governing development length are essentially the same as those for splice length.

KEY WORDS: lap splices, deformed bars, test, beams.

IMPLEMENTATION

The design proposals made in this study are based on equations derived empirically using test results from a number of well-documented studies. The basic equation proposed for splice or development length is a function of steel stress, concrete strength, bar diameter, side or bottom cover and transverse reinforcement, is expressed as follows:

For Grade 60 reinforcement

$$\ell_s \text{ or } \ell_d = \frac{10200\, d_b}{\sqrt{f'_c}\, \varphi(1 + 2.5C/d_b + K_{tr})}$$

It is recommended that the value of C/d_b to be used in this equation be not more than 2.5 and the resulting ℓ_s or ℓ_d be not less than 12 in. The factor K_{tr} represents the effect of transverse reinforcement. A capacity reduction factor φ of 0.8 is recommended. Modification factors for other grade steels, for wide spacings, and for top cast bars are presented.

The use of the proposed design can produce splices as much as 60 percent shorter than those designed under current AASHTO provisions. Such changes can materially reduce the congestion in spliced regions of reinforced concrete members and simplify construction procedures. In addition, the proposed design approach consolidates development and splice length provisions under a single specification which is convenient to use and interpret. Therefore, the implementation of the proposed design should result in substantial economies in design time and material costs.

CONTENTS

LIST OF TABLES

Table | Page

LIST OF FIGURES

N O T A T I O N S

The following notations have been used in this report.

a_b	area of bar
A_{tr}	area of transverse reinforcement normal to the plane of splitting through the anchored bars
C	the smaller of C_b or C_s
C_b	clear bottom cover to main reinforcement
C_s	half clear spacing between bars or splices or half available concrete width per bar or splice resisting splitting in the failure plane
d_b	diameter of main reinforcement
f'_c	concrete cylinder strength
f_s	maximum stress in bar
f'_t	concrete tensile strength, taken as proportional to $\sqrt{f'_c}$
f_{yt}	yield strength of transverse reinforcement
k	ratio of steel stresses
K_{tr}	an index of the transverse reinforcement provided along the anchored bar, $A_{tr}f_{yt}/500\ sd_b$
ℓ_d	development length
ℓ_s	length of lap splice
s	spacing of transverse reinforcement, center to center
S'	clear splice spacing, laterally
u	average bond
u_c	portion of strength contributed by concrete cover
u_{cal}	calculated average bond stress
u_t	average bond stress obtained in tests
u_{tr}	portion of strength contributed by transverse reinforcement

1. INTRODUCTION

The design of lap splices in reinforced concrete structures is of continuing interest to structural engineers because of the implications of splice length on detailing and on structural performance. The design of splices in highway structures is governed by the 1974 AASHTO Interim Specifications for Bridges. The AASHTO Specifications have been adopted from the 1971 ACI Building Code Requirements for Reinforced Concrete (ACI 318-71). The appropriate sections of the AASHTO Specifications are repeated below.

1.1 AASHTO Specifications for Tension Splices

The following sections have been extracted directly from the 1974 AASHTO Interim Specifications for Bridges:

1.5.22--SPLICES IN REINFORCEMENT

(A) General

(1) Splices of reinforcement shall be made only as shown on the design drawings or as specified, or as authorized by the engineer. Except as provided herein, all welding shall conform to Recommended Practices for Welding Reinforcing Steel, Metal Inserts and Connections in Reinforced Concrete Construction (AWS D12.1).

(2) Lap splices shall not be used for bars larger than No. 11.

(3) Lap splices of bundled bars shall be based on the lap splice length required for individual bars of the same size as the bars spliced and such individual splices within the bundle shall not overlap each other. The length of lap as prescribed in Article 1.5.22(B) or (C) shall be increased 20 percent for a three-bar bundle and 33 percent for a four-bar bundle.

(4) Bars spliced by noncontact lap splices in flexural members shall not be spaced transversely farther apart than one-fifth the required length of lap nor 6 in.

(5) Welded splices or other positive connections may be used. A full welded splice is one in which the bars are butted and welded to develop in tension at least 125 percent of the specified yield strength of the bar.

A full positive connection is one in which the bars are connected to develop in tension or compression, as required, at least 125 percent of the specified yield strength of the bar.

(B) Splices in Tension

(1) Classification of tension lap splices--The minimum length of lap for tension lap splices shall be that given in this Article, but not less than 12 inches. ℓ_d is the tensile development length for the full f_y as given in Article 1.5.14(1), (2), (3) and (4).

Class A splices	$1.0\ell_d$
Class B splices	$1.3\ell_d$
Class C splices	$1.7\ell_d$
Class D splices	$2.0\ell_d$

The bars in a Class D splice shall be enclosed within a spiral meeting the requirements of Article 1.5.14(4) but no reduction in required development length shall be allowed for the effect of the spiral. In a Class D splice the ends of bars larger than No. 4 shall be hooked 180-deg.

(2) Splices in tension tie members--Where feasible, splices shall be staggered and made with full welded or full positive connections as given in Article 1.5.22(A)(5). If lap splices are used, they shall meet the requirements of a Class D splice (lap of $2.0\ell_d$).

(3) Tension splices in other members--

(a) In regions of high tensile stress--Splices in regions where the tensile reinforcement provided is equal to or less than twice that required for strength shall meet the following requirements:

If no more than one-half the bars are lap spliced within a required lap length, splices shall meet the requirements for Class B splices (lap of $1.3\ell_d$).

If more than one-half of the bars are lap spliced within a required lap length, splices shall meet the requirements for Class C splices (lap of $1.7\ell_d$).

If welded splices or positive connections are used they shall meet the requirements of Article 1.5.22(A)(5).

(b) In regions of low tensile stress--Splices in regions where the tensile reinforcement provided is more than twice that required for strength shall meet the following requirements:

If no more than three-quarters of the bars are lap spliced within a required lap length, splices shall meet the requirements for Class A splices (lap of $1.0\ell_d$).

If more than three-quarters of the bars are lap spliced within a required lap length, splices shall meet the requirements for Class B splices (lap of $1.3\ell_d$).

If welded splices or positive connections are used, the requirements of Article 1.5.22(A)(5) may be waived if the splices are staggered at least 24 in. and in such a manner as to develop at every section at least twice the calculated tensile force at the section and in no case less than 20,000 psi on the total sectional area of all bars used. In computing the capacity developed at each section, spliced bars shall be rated at the specified splice strength. Unspliced bars shall be rated at the amount of anchorage provided on either side of the section.

1.5.14--DEVELOPMENT LENGTH OF DEFORMED BARS AND DEFORMED WIRE IN TENSION

The development length ℓ_d, in inches, of deformed bars and deformed wire in tension shall be computed as the product of the basic development length of (1) and the applicable modification factor or factors of (2), (3), and (4), but ℓ_d shall be not less than that specified in (5).

(1) The basic development length shall be:

For #11 or smaller bars. . . . . $0.04A_b f_y/(f'_c)^{1/2}$ [1]

but not less than. . . . . $0.0004d_b f_y$ [2]

For #14 bars . . . . $0.085f_y/(f'_c)^{1/2}$ [3]

For #18 bars . . . . $0.11f_y/(f'_c)^{1/2}$ [3]

For deformed wire. . . . . $0.03d_b f_y/(f'_c)^{1/2}$

(2) The basic development length shall be multiplied by a factor of 1.4 for top reinforcement.[4]

(3) When lightweight aggregate concrete is used, the basic development lengths in (1) shall be multiplied by 1.33 for "all-lightweight" concrete and 1.18 for "sand-lightweight" concrete with linear interpolation when partial sand replacement is used, or the basic development length may be multiplied by $6.7(f'_c)^{1/2}/f_{ct}$, but not less than 1.0 when f_{ct} is specified. The factors of (2) and (4) shall also be applied.

(4) The basic development length may be multiplied by the applicable factor or factors for:

Reinforcement being developed in the length under consideration and spaced laterally at least 6 in. on center and at least 3 in. from the side face of the member 0.8

Where anchorage or development for f_y is not specifically required, reinforcement in flexural members in excess of that required . . . . . . (A_s required)/(A_s provided)

Bars enclosed within a spiral which is not less than ¼ in. diameter and not more than 4 in. pitch 0.75

(5) The development length, ℓ_d, shall be taken as not less than 12 in. except in the computation of lap splices by Article 1.5.22(B) and anchorage of shear reinforcement by Article 1.5.21.

---

[1] The constant carries the unit of 1/in.

[2] The constant carries the unit of $in.^2/lb$.

[3] The constant carries the unit of in.

[4] Top reinforcement is horizontal reinforcement so placed that more than 12 in. of concrete is cast in the member below the bar.

1.2 Background of Current Specifications

In order to discuss the applicability of current design provisions it is useful to examine briefly the basis on which the specifications were developed. Splice lengths are currently based on the development length ℓ_d. Depending on the severity of stresses, the splice length is increased. For example, if more than 50 percent of the bars are spliced in the region of maximum stress ($f_s > 0.5f_y$), the splice length $\ell_s = 1.7\ell_d$. The basic premise is that the cover on the bar may be at a minimum value and that the splice should develop at least 25 percent more stress than computed from a consideration of moments at the splice region.

It should be noted that development lengths ℓ_d in ACI 318-71 are based on ultimate bond stresses specified in ACI 318-63. Ultimate bond stress for bottom bars was a function of concrete strength f'_c and bar diameter d_b as follows:

$$u_u = \frac{9.5\sqrt{f'_c}}{d_b} \leq 800 \text{ psi}$$

Assuming a uniform distribution of bond stress along a bar with area a_b, the length needed to develop 125 percent of yield is determined in the following manner. Equating the tensile force on the bar with the total bond force on the surface area of the bar yields

$$\ell_d \pi d_b u_u = a_b(1.25f_y)$$

from which the equation for ℓ_d in ACI 318-71 is derived.

$$\ell_d = \frac{a_b(1.25f_y)}{\pi d_b(9.5\sqrt{f'_c}/d_b)} \approx 0.04a_b f_y/\sqrt{f'_c} \qquad (1)$$

No φ factor was specified for development length computations because the area of steel provided at a section was based on a $\varphi = 0.9$ (flexural reinforcement). Therefore, it was not felt necessary to include a φ factor for development length considering that a φ of 0.9 was already included in determining steel areas and, in addition, the length was based on assuming that the steel develops $1.25f_y$.

Furthermore, it is important to note that the data available regarding the strength of lapped splices was limited at the time the current provisions were developed. Therefore, a reevaluation of design specifications for splices and development lengths considering recent test data is needed.

2. A FAILURE HYPOTHESIS FOR ANCHORED BARS

2.1 Stress Transfer between Reinforcing Bars in Concrete

The transfer of stress from a deformed bar to the concrete is accomplished mainly by mechanical locking of the lugs into the surrounding concrete. The resultant force exerted by the lug on the concrete is inclined at an angle β to the axis of the bar (Fig. 1) and it is the radial component that is the cause of splitting of the surrounding concrete at failure. If the stress component parallel to the axis of the bar is u, the radial stress component of the bond force is u tan β. The radial stress can be regarded as a water pressure acting against a thick-walled cylinder having an inner diameter equal to the bar diameter and a thickness C the smaller of (1) the clear bottom cover C_b, or (2) half the clear spacing C_s between the next adjacent bar (see Fig. 2). The load-carrying capacity of the cylinder depends on the tensile strength of the concrete. When this is exhausted, splitting cracks form in the concrete. With $C_b > C_s$, a horizontal split develops at the level of the bars, and is termed a <u>side split failure</u>. With $C_s > C_b$, longitudinal cracks through the bottom cover form before the occurrence of splitting along the plane of the bars. Such a failure is termed a <u>face-and-side split failure</u>. With $C_s \gg C_b$, the longitudinal cracks form prior to inclined cracks which form a <u>V-notch failure</u>. The splitting patterns in Fig. 2 correspond to those described in a report[17] by ACI Committee 408--Bond Stress.

In a lap splice where the bars are laid side by side, the two cylinders to be considered for each splice interact to form, in section, an oval ring, as shown in Fig. 3. The failure patterns are similar to those of single bars. The side split failure results for $C_b > C_s$, the face-and-side split failure failure for $C_s > C_b$, and the V-notch failure for $C_s \gg C_b$.

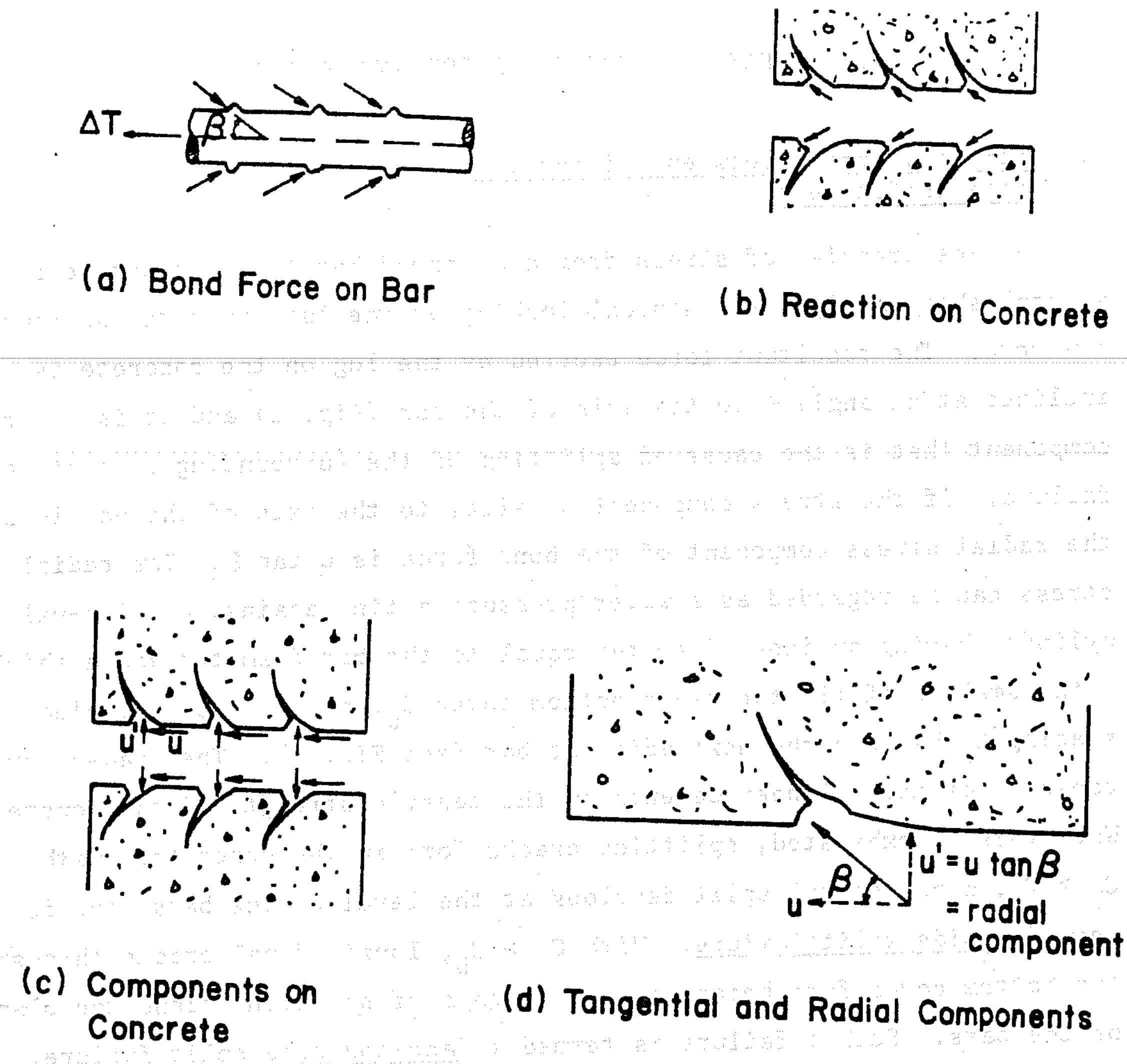


Fig. 1. Forces between deformed bar and concrete.

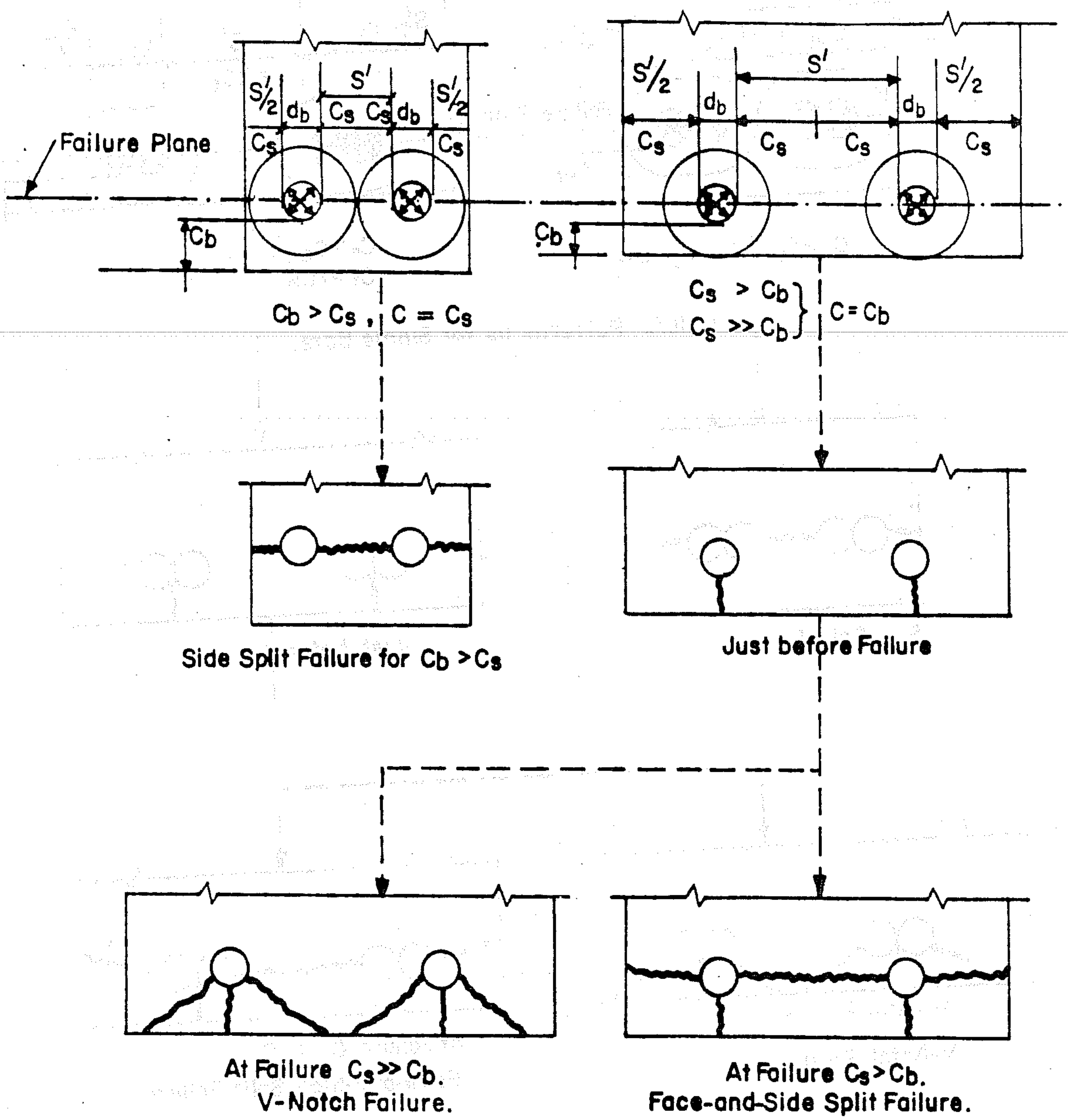


Fig 2. Failure patterns of deformed bars.

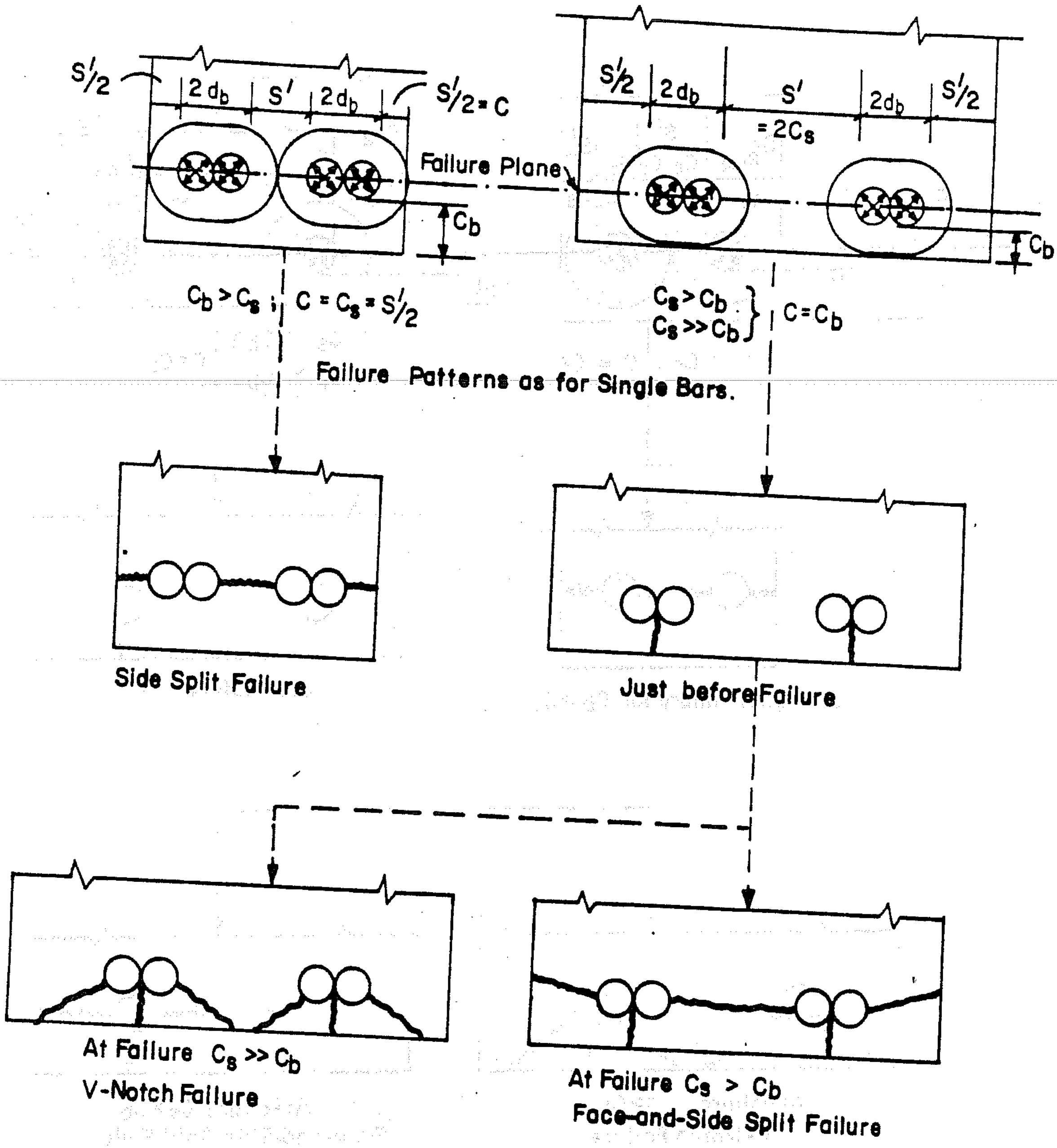


Fig 3. Failure patterns in lapped splices.

It is possible with the water pressure analogy to analyze the stress in a concrete cylinder surrounding a single bar and this has been done by Tepfers.[6] No attempt has yet been made to analyze the stresses in the concrete cylinder having an oval ring cross section surrounding two bars laid side by side, as in Fig. 3. Such a solution is likely to be complex. The uneven distribution of bond stress and the uncertainty in the value of β may lead to further complications.

Measurement of bar strains along lap splices by Ferguson and Briceno[1] and also by Tepfers[6] shows that the strain variation along the splice becomes approximately linear near the ultimate load. Therefore, the tangential stress, u, is constant and can be determined from the maximum stress in the bar, i.e., $u = d_b f_s / 4\ell_s$. Consequently, if the value of β is known, it is possible to determine the radial force causing splitting in the failure plane. By equating the tensile resistance of concrete to the splitting forces, a relationship between material and geometrical properties of the splice section can be determined. From measurement of slopes of internal cracks radiating from a tension bar embedded in concrete prism in an experiment by Goto,[18] it was found that the angle of inclination of the force can vary from 45° to 80° and depends on whether the ribs are lateral, diagonal, or wavy with respect to the axis of the bar.

Equating concrete tensile resistance with splitting forces, Ferguson and Briceno[1] developed equations for side split and face-and-side split failures. The assumption was made that radial and longitudinal components of force between the bar and concrete are equal ($\beta = 45^\circ$). It should be noted that splitting was assumed to occur instantaneously along the entire splice; however, splitting would actually be progressive starting at the end of the splice. Although the values of f_t' obtained from the analysis compared well with split cylinder test values, the equations obtained are rather complex for design.

Ferguson and Krishnaswamy[2] used a slightly different approach to evaluate the relationship between tensile resistance of the concrete to splitting and bar force. It was assumed that the splitting force is related to bar force but may not be equal to it (i.e., β may be more or less than 45^{o}). An equation was developed relating the computed average tensile stress in the concrete f_{tu} to f'_t, the concrete tensile strength. The tensile force in the concrete over the length of the splice can be expressed as $f_{tu}S'\ell_s$. The component of the force normal to the plane of splitting is $f_s(\pi d_b^2/4)\tan\beta$. For cases where a moment gradient is present along the splice, the average stress at the two ends is used or $f_s(1 + k)/2$, where k is the ratio of lower to higher steel stresses at the splice ends. Equating the splitting force to the component of bar force yields the following expression:

$$f_{tu}S'\ell_s = f_s\left(\frac{1+k}{2}\right)\left(\frac{\pi\, d_b^2}{4}\right)\tan\beta$$

Substituting average bond stress $u = d_b f_s/4\ell_s$ and rearranging gives

$$f_{tu} = \frac{u\, d_b(1+k)}{S'}\left(\frac{\pi\tan\beta}{2}\right)$$

Therefore, the ratio f_{tu}/f'_t can be expressed as follows:

$$\alpha = \frac{f_{tu}}{f'_t} = \frac{u\, d_b(1+k)}{f'_t S'} \tag{2}$$

with the unknown $\tan\beta$ incorporated into α. Ferguson and Krishnaswamy took $f'_t = 6.4\sqrt{f'_c}$, a value based on split cylinder tests. Using data from tests conducted at The University of Texas, values of α were computed. A plot of α versus S'/C_b is shown in Fig. 4. From these data a relationship between $1/\alpha$ and S'/C_b was derived and used to develop a design equation for splice length. For 3000 psi concrete and Grade 60 reinforcement developing $1.25f_y$ for ductility, the equation is

$$\ell_s = 100d_b^2(1/S' + 1/2C_b) \tag{3}$$

Some additional modifications were suggested for transverse reinforcement, for $C_b > S'$, for top cast or lightweight concrete, for interior splices, and for a moment gradient along the splice.

The possibility of determining a mean value for β from test results on development lengths by using a relationship derived by Tepfers[6] was investigated in this study. In deriving the relationship, Tepfers assumed

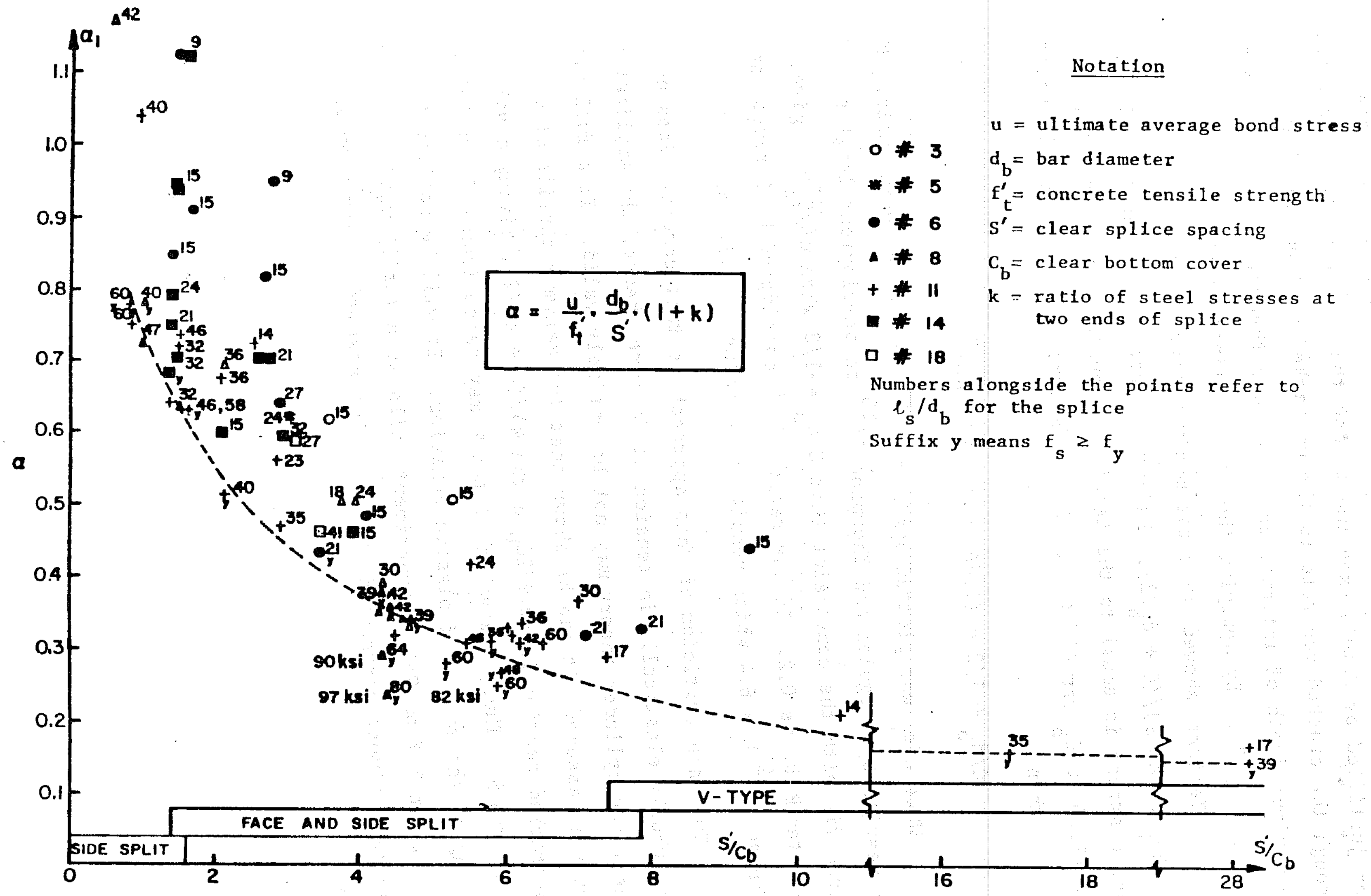


Fig 4. Variation of α with S'/C_b.

that the concrete around a deformed bar in tension is cracked--an assumption justified by Goto's experiment--and that the bond force is carried through the cracked concrete to the uncracked section, as shown in Fig. 5. The maximum depth of internal crack, e, was theoretically shown to be $0.486(C + d_b/2)$. By applying the thick cylinder theory to the uncracked section, Tepfers showed that the maximum tensile stress is $(1.664 d_b \; u \tan \beta)/(C + d_b/2)$. Failure occurs as soon as this maximum tensile stress is equal to the tensile strength of the concrete, i.e., $f'_t = (1.664 \; u \; d_b \tan \beta)/(C + d_b/2)$ at failure. Since f'_t can be written as $k_1 \sqrt{f'_c}$, then

$$C/d_b + 1/2 = (1.664 \; u \tan \beta)/(k_1 \sqrt{f'_c}) \tag{4}$$

When C/d_b was plotted against $u/\sqrt{f'_c}$ in Fig. 6, using mainly the test results by Ferguson and Thompson[12,13] on development lengths, a least squares fit with the constraint that $C/d_b = -1/2$ when $u/\sqrt{f'_c} = 0$ gives $(1.664 \tan \beta) k_1$ as 0.2. In the range of f'_c considered by Ferguson and Krishnaswamy,[2] $k_1 = 6.4$ which results in a value of 0.77 for $\tan \beta$.

The main criticism of this approach is that concrete does not behave wholly elastically in tension at failure; hence, the application of the thick cylinder theory may not be entirely valid. If a full plastic behavior is assumed, it can be shown that the maximum tensile stress in the uncracked section is $(0.972 d_b \; u \tan \beta)/(C + d_b/2)$, giving a value of 1.32 for $\tan \beta$. Thus, the value of $\tan \beta$ may range from 0.77 to 1.32, depending on the extent of plastic behavior. It will be noticed that values of $\tan \beta$ from Goto's experiment falls essentially within this range and the mean almost corresponds to the value assumed by Ferguson and Briceno.[1]

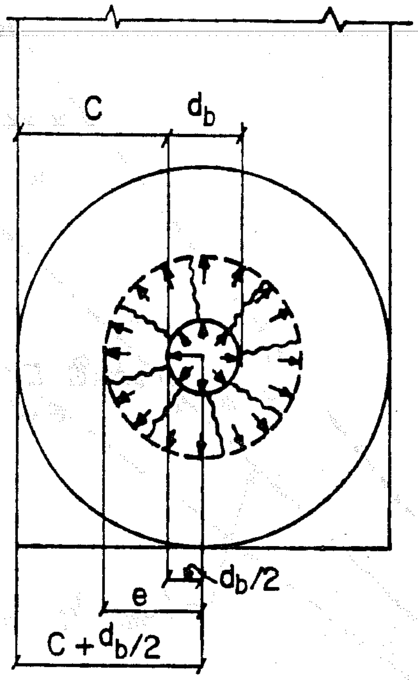


Fig. 5. Internal cracks surrounding a deformed bar in concrete.

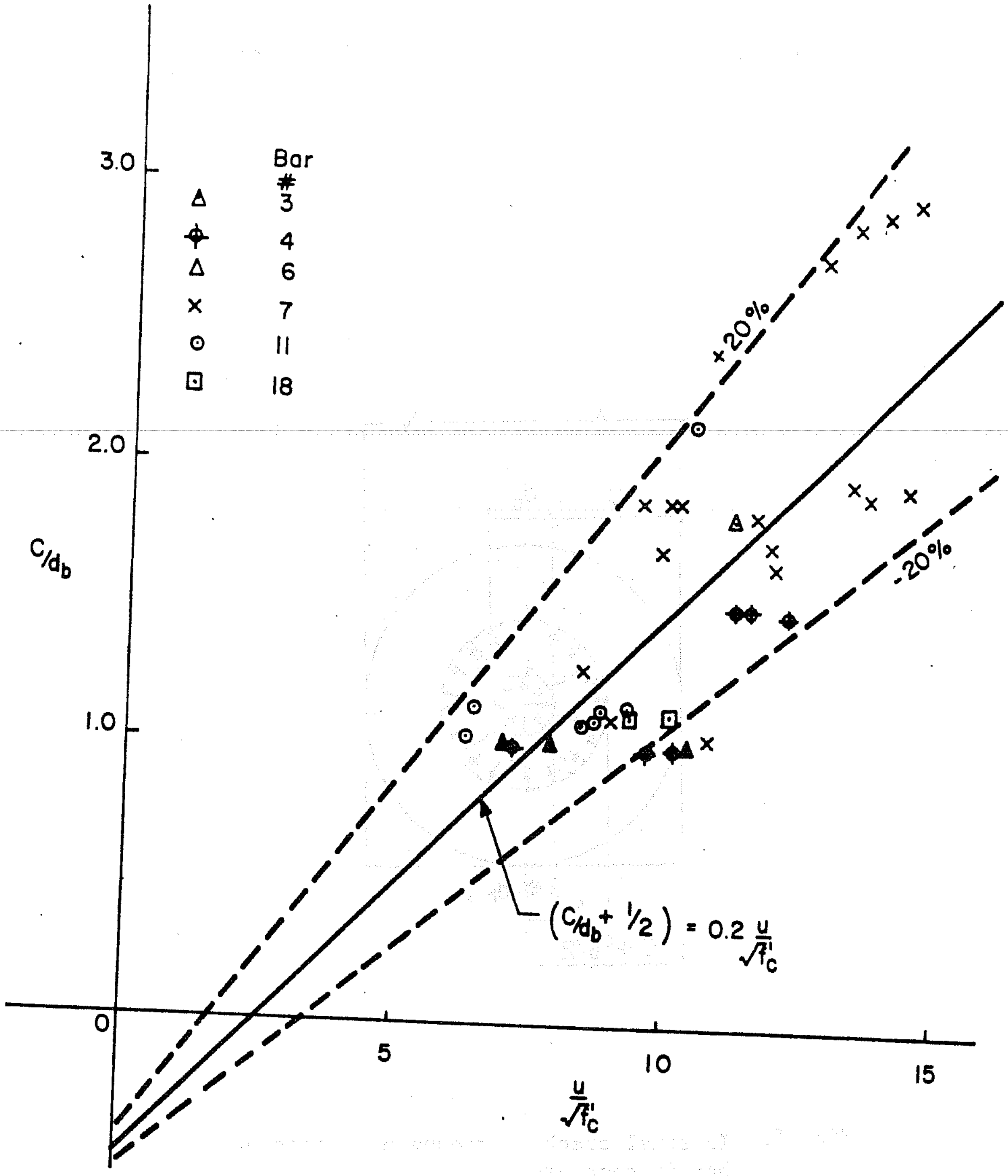


Fig 6. Variation of $u/\sqrt{f'_c}$ with C/d_b in development length tests.

3. BARS WITHOUT TRANSVERSE REINFORCEMENT

3.1 Influence of Cover and Spacing

Since the value of β can vary substantially depending on the assumptions made, it was decided to give up a theoretical approach in favor of an empirical one. In the following analysis, the strength of a lap splice at failure is related to an average bond stress u, determined from the maximum steel stress reached, i.e., $u = d_b f_s / 4\ell_s$. It is assumed that the failure of the splice occurs following the appearance of cracks either at the sides or on the tension face (Fig. 3). This reduces to one parameter the influence of cover and spacing and is an essential departure from the empirical approach by Ferguson and Krishnaswamy,[2] where both bottom cover and side spacing were considered as separate parameters. The assumption is valid for $C_b > C_s$, but should lead to conservative values for wide spacing because of the contribution to tensile strength in the failure plane by the concrete outside the oval ring considered. As the contribution is not directly proportional to side spacing, clear cover and side spacing are not considered as separate parameters. The effect of wide spacing is further discussed later.

3.2 Formulation of Equation--Splice Tests

Test results indicate that the average bond stress, u, for a lap splice in a constant moment region and without transverse reinforcement depends on

(1) the tensile strength of the concrete
(2) the cover C as defined in Fig. 3
(3) the diameter d_b of the bar
(4) the length of the splice ℓ_s

The variables u, f_t', C, d_b, and ℓ_s can be arranged to form dimensionless parameters u/f_t', C/d_b, and d_b/ℓ_s, and from dimensional analysis u/f_t' is a function of $(C/d_b, d_b/\ell_s)$. The concrete tensile strength f'_t is usually taken as proportional to $\sqrt{f'_c}$, so that $u/\sqrt{f'_c}$ is a function of $(C/d_b, d_b/\ell_s)$. Bond tests by Mathey and Watstein[16] indicated that u varies approximately linearly with d_b/ℓ_s. Various functions were investigated with the aim of retaining a simple equation for conversion to a design provision. The three equations below appeared to be most promising.

(a) $u/\sqrt{f'_c} = b_1 + b_2(C/d_b)^2 + b_3C/d_b + b_4d_b/\ell_s$

(b) $u/\sqrt{f'_c} = b_1 + b_2(C/d_b)^2 + b_3d_b/\ell_s$

(c) $u/\sqrt{f'_c} = b_1 + b_2C/d_b + b_3d_b/\ell_s$

The constants b_1, b_2, b_3, and b_4 were determined from a nonlinear regression analysis of test results of 62 beams tabulated in Table 1* which were tested by Chinn, Ferguson, and Thompson,[3] Ferguson and Breen,[4] Chamberlin,[5] and Ferguson and Krishnaswamy.[2] The beams had one or two splices with the bars in contact and all the bars were spliced at the same section. All the beams were tested in flexure with constant moment all through the splice length. Further particulars of the test specimens are given in Figs. 7 and 8. Only specimens in which the steel did not reach yield were included. It was felt that the bar elongations produced by yielding may produce failures which would not occur if the bar is in the elastic range when splitting occurs. The standard error of estimate was 1.259 for (a), 1.280 for (b), and 1.278 for (c). Since the standard errors of estimate were almost equal, the simplest function (c) was chosen. The regression analysis gave the following values for the constants.

$$u^*/\sqrt{f'_c} = 1.22 + 3.23\ C/d_b + 53.0d_b/\ell_s \qquad (5)$$

where u* denotes the selected best fit equation for beams with constant moment over the splice length.

The measured bond stresses $[u_t = f_s(\text{measured}) \times d_b/4\ell_s]$ divided by $\sqrt{f'_c}$ are plotted against ℓ_s/d_b in Figs. 9, 10, and 11. The test results are grouped according to C/d_b ratios and in each figure Eq. (5) is shown for the average C/d_b ratio of the tests plotted. The coefficients in Eq. (5) were rounded off and the resulting Eq. (6), which yields values

---

*All tables are in Appendix A.

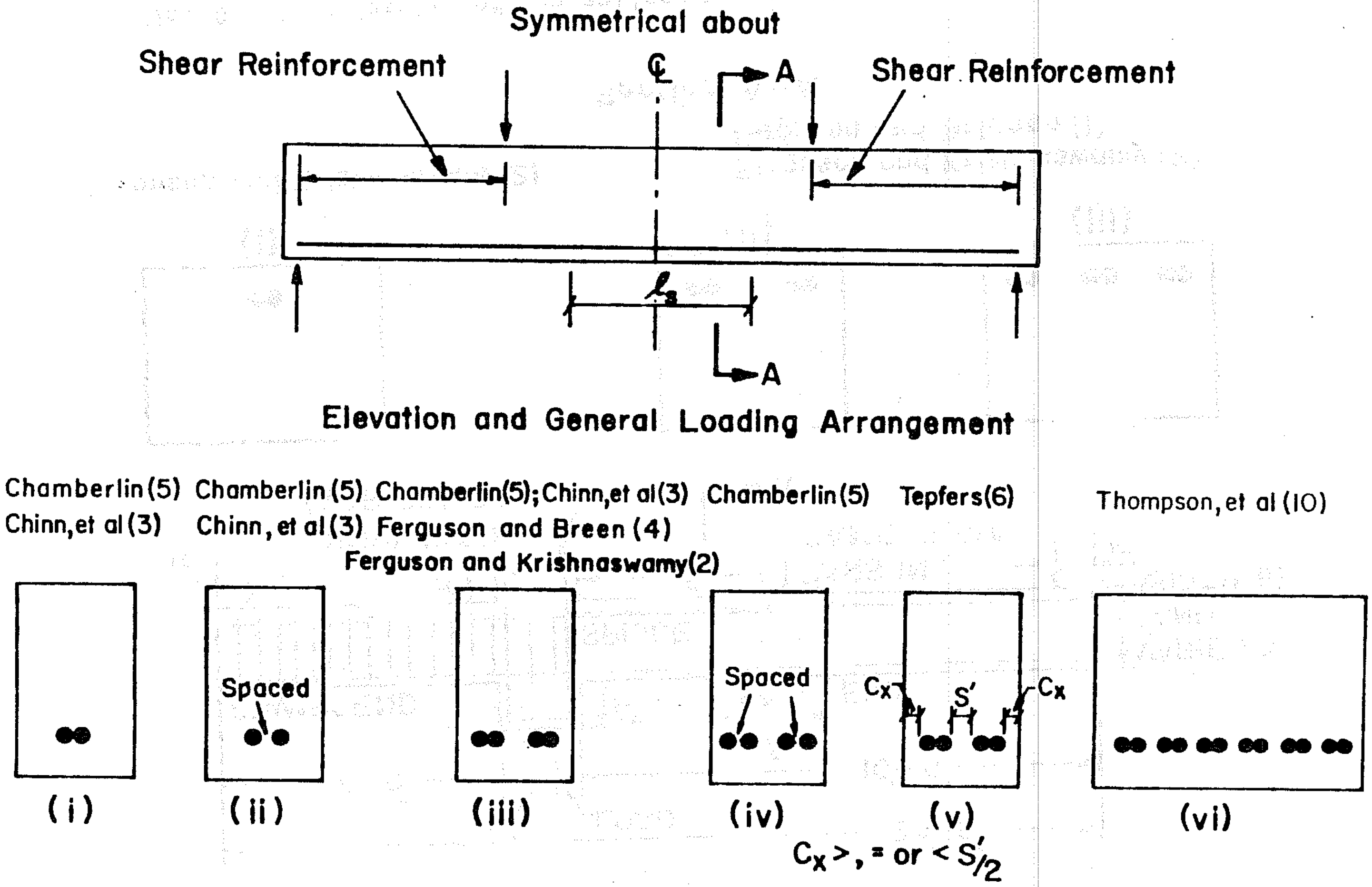


Fig. 7. Test details--lap splices without transverse reinforcement.

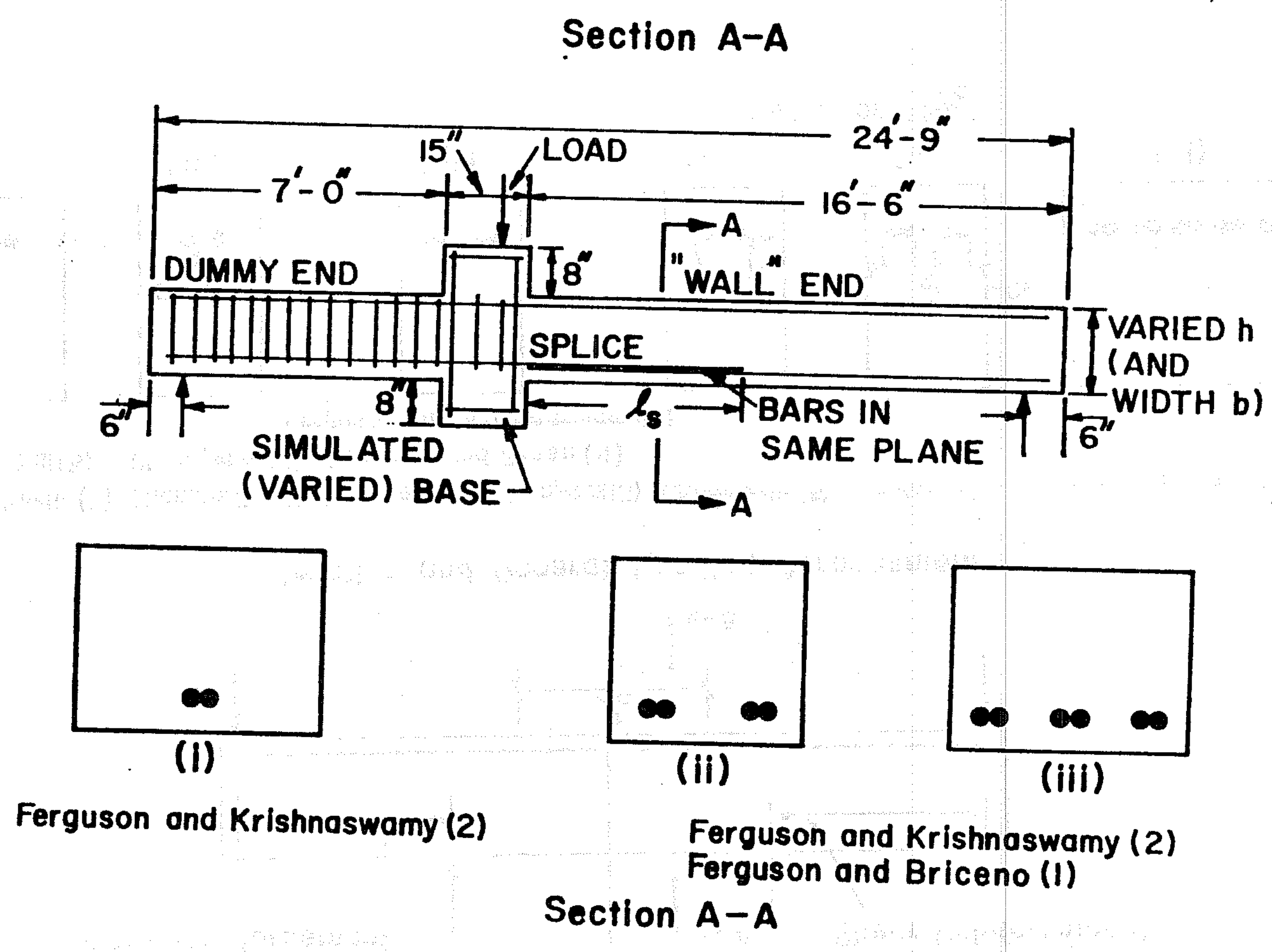


Fig. 8. Test details of lap splices without transverse reinforcement.

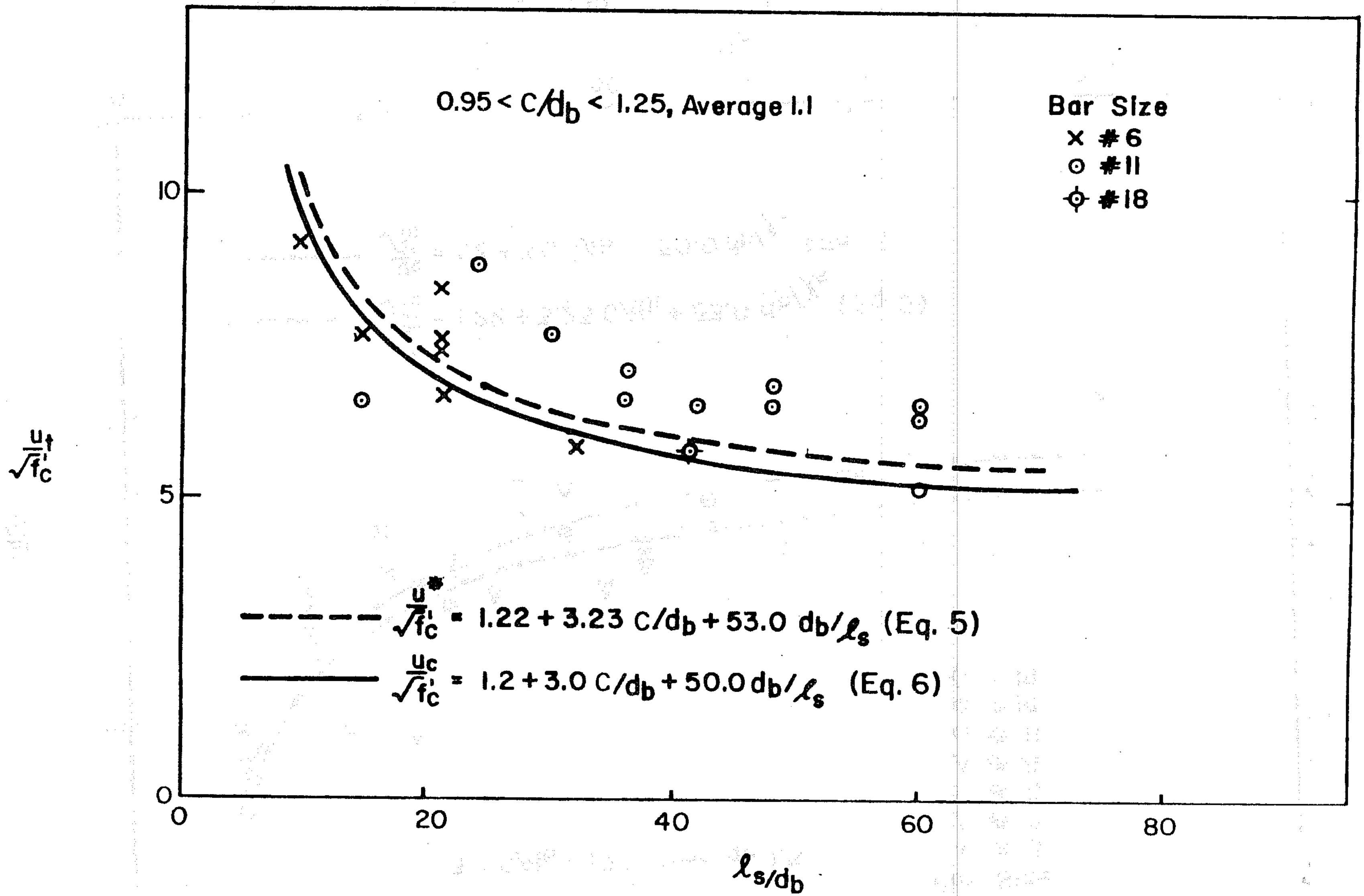


Fig. 9. Variation of $u_t/\sqrt{f'_c}$ with ℓ_s/d_b at an average C/d_b of 1.1.

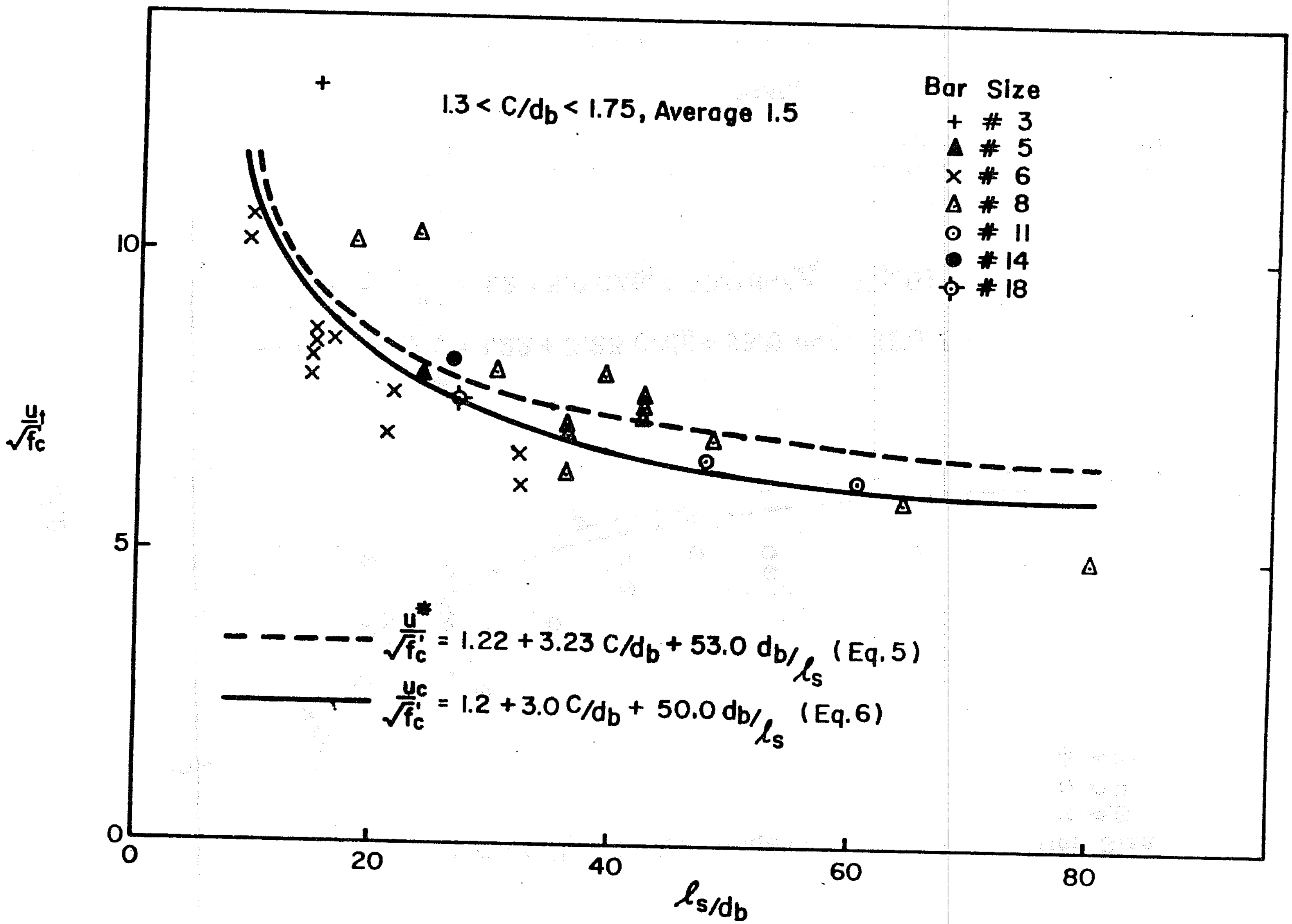


Fig. 10. Variation of $u_t/\sqrt{f'_c}$ with ℓ_s/d_b at an average C/d_b of 1.5.

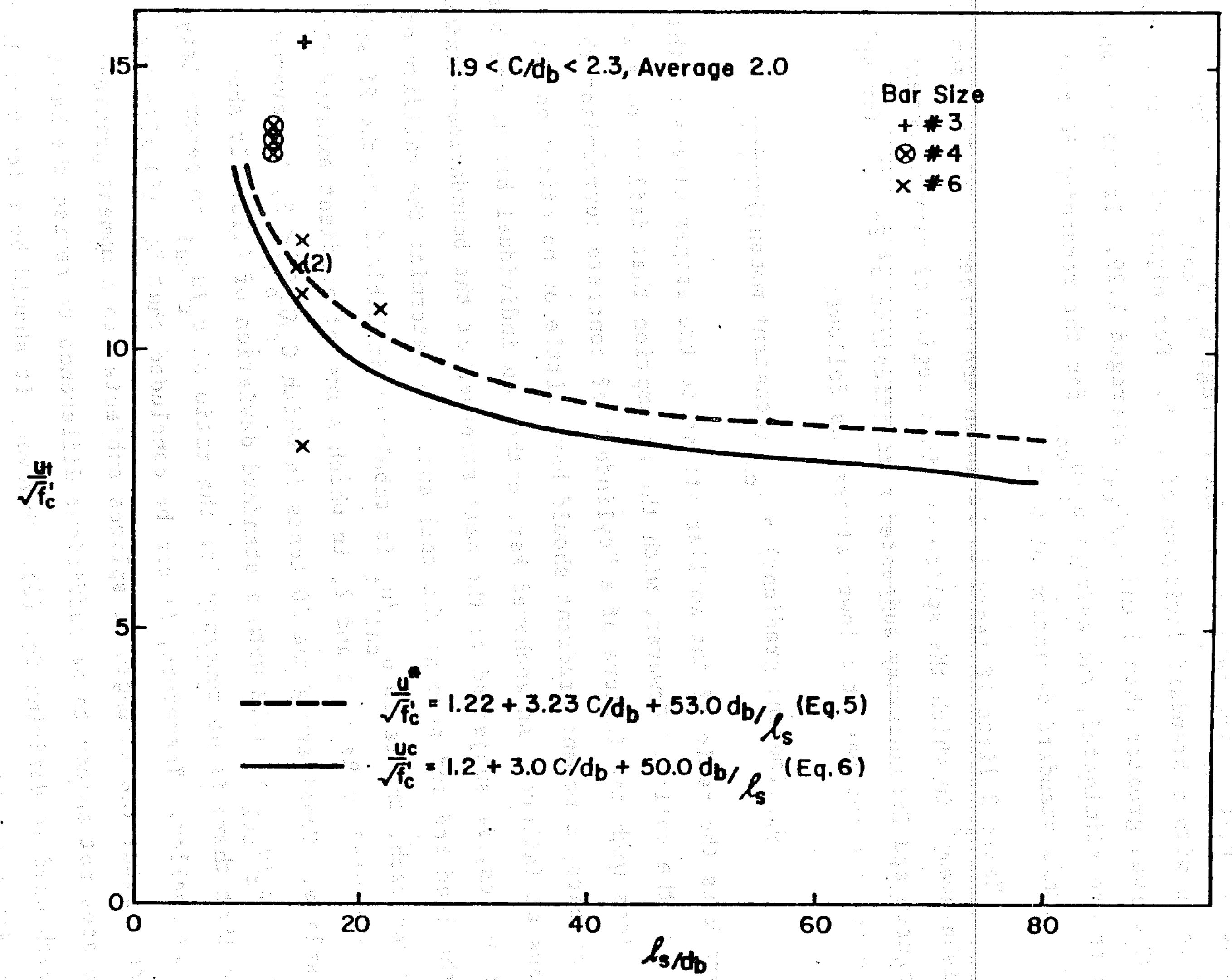


Fig 11. Variation of $u_t/\sqrt{f'_c}$ with ℓ_s/d_b at an average C/d_b of 2.0.

of u slightly lower than Eq. (5) is also plotted in Figs. 9, 10, and 11.

$$u_c/\sqrt{f'_c} = 1.2 + 3C/d_b + 50d_b/\ell_s \tag{6}$$

Values of the stress calculated using Eq. (6) u_{cal} are listed in Table 1 and ratios of u_t/u_{cal} are tabulated. The average $u_t/u_{cal} = 1.07$ for all 62 tests with a standard deviation of 0.15. For eight of the tests the $C_s/(C_b d_b)$ was greater than 3 and u_t/u_{cal} averaged 1.29. If these eight tests are eliminated, the average u_t/u_{cal} for the remaining 54 tests is 1.03 with a standard deviation of 0.12.

Table 2 lists 28 tests by Ferguson and Briceno[1] and Ferguson and Krishnaswamy[2] in which the splice was in a region of varying moment. Ferguson and Krishnaswamy suggested a modification of Eq. (3) for splices in which one end was at a lower stress, as follows:

$$u_{cal}(\text{moment gradient}) = u_{cal}(\text{constant moment})\frac{2}{(1 + k)}$$

where k is the ratio of the smaller stress to the larger stress at the two ends of the splice. However, with the assumption that failure of a splice coincides with the failure of a "cylinder" of concrete surrounding the bar or bars, a moment gradient should have little or no effect on the stress at failure. An anchored bar, either an individual bar or one bar in a splice, is subjected to the same stresses at the boundaries--maximum at the lead end and zero at the tail end. To determine the validity of this approach, the ratio u_{cal}/u_t is tabulated in Table 2 for the 28 splice tests reported in Refs. 1 and 2, in which a moment gradient existed along the splice. Considering the 20 tests in which $C_s/(C_b d_b) < 3$, the average value of u_t/u_{cal} is 1.12 with a standard deviation of 0.13. It should be noted that there is no tendency for the ratio of u_t/u_{cal} to become large as k is smaller. Therefore, it can be concluded that Eq. (6) slightly underestimates the strength of splices subjected to a moment gradient. There does not appear to be sufficient difference to revise the basic approach used in deriving Eq. (6). However, it should be noted that in the tests with the splice in the region of variable moment the splices were subjected to a fairly low constant shear force. A splice may not perform as well in a region of high, varying shear.

3.3 Other Splice Tests--No Transverse Reinforcement

A number of additional splice tests reported in the literature were omitted in the initial development of the empirical equation for for average bond stress and these are listed in Tables 3 and 4.

Nine tests reported in Refs. 3 and 5 were omitted because the spliced bars were not in contact but had variable spacings between them. In these tests C_s is taken as half the total net concrete width resisting splitting in the plane of the bars divided by the number of splices. Ferguson, Turpin, and Thompson[7] showed that for a given overall width of specimen the strength of a bar is essentially the same if the bar is located concentrically or is displaced off the center. Table 3 also lists the results of a series of wide specimens containing five or six spliced bars which simulate a retaining wall reported by Thompson, et al.[10] The purpose of the tests was to determine whether the outside or edge splice initiates failure of the specimen. In most tests the stress in the edge splices was less than in the interior splices. Table 3 includes average values of u_t for all splices in the section as well as u_t for the edge splices. The ratio of u_t/u_{cal} is shown for both conditions. Considering all splices in the section average u_t/u_{cal} is 1.13 and for the edge splices u_t/u_{cal} averages 0.97.

A major study of splices was reported by Tepfers.[6] The test specimen is shown in Fig. 7. Because the bars may have deformations which are not comparable with those used in the U.S., the data were not included in the initial development of the empirical equation [Eq. (6)]. The results are listed in Table 4. Dimensions are listed in metric units, since Eq. (6) utilizes ratios of dimensions. The 6 in. cube strengths reported by Tepfers were converted to cylinder strengths using a factor of 0.81 suggested by Neville.[8] The average u_t/u_{cal} was 1.18 for the 92 splice tests with no transverse reinforcement and the standard deviation was 0.32. While the correlation between computed and measured stresses was not as close for Tepfers' tests as for the other tests reported here, it should be remembered that the deformed bars may be different from those used in the U.S. and concrete strengths were reported for cubes and required conversion to cylinder strength for use in the equation.

3.4 Limitation on Influence of Cover

In Eq. (6) the strength of the bar increases as the cover to bar diameter ratio increases. However, it is obvious that at some cover to diameter ratio the mode of failure will not involve splitting. For large C/d_b values, direct pull-out could occur with the bar deformation shearing off the concrete in between the lugs. Since most of the data on which the empirical equation is based are limited to C/d_b ratios of 2.5 or less, it is suggested that C/d_b be limited to 2.5 in Eq. (6). However, the actual values of C/d_b have been used to determine u_{cal} in Tables 1-4 in the Appendix.

3.5 Effect of Staggering Splices

Codes of practice favor staggering splices with respect to each other in the longitudinal direction. Such practice has been shown[15] to reduce the width of flexural cracks at ends of lap splices. Test data are available only for seven beams to check the effect of staggering splices. Three of the tests had one of the reinforcing bars continuous, while the other is spliced (i.e., 50 percent of reinforcement spliced), and three had 67 percent of the reinforcement spliced at one section. The remaining test had two splices staggered with respect to each other. The results of these tests indicated improved strength in comparison with other tests with 100 percent of the reinforcement spliced at one section. Until further tests quantify the effect of staggering splices, it is recommended that in cases where alternate splices are staggered by at least one-half the splice length, the side cover can be determined by ignoring the adjacent continuous bar at the critical section through the end of the splice.

3.6 Splices in Retaining Walls

A study of the behavior of splices in retaining walls was conducted by Thompson et al. and is reported in Ref. 10. Previous studies had indicated that there was a tendency for failure of a specimen to be initiated by the edge splice. For the tests reported in Ref. 10, which contained five to six spliced bars (see Fig. 7), there was evidence of splitting starting at the edge splice. However, the difference in the stresses between edge and interior splices at failure was generally less than 15 percent. Ratios of u_t/u_{cal} for edge and interior splices are listed in Table 3. The ratios of u_t/u_{cal} for edge splices averaged about 0.97. On this basis there does not appear to be a need to modify the equation developed for interior splices. The slightly higher strength of interior splices simply serves as an added factor of safety in a retaining wall which has no redundancy and depends entirely on the splice for strength.

3.7 Splices under Impact Loads

A study of lapped splices under rapid impact loading is reported in Ref. 22. The specimens contained two spliced #8 bars and were subjected to a number of different loading conditions, including single loading to failure, incrementally increasing loads to failure, repeated loads, and repeated reversed loads. The objective of the study was to determine whether splice length provisions based on static test results were adequate if impact or dynamic loads were imposed. The results indicate that splice lengths, calculated using provisions based on static tests, are satisfactory if subjected to impact loadings.

3.8 Application to Development Lengths

Similar behavior in cracking and splitting has been observed in tests for development lengths and lap splices. As shown in Figs. 2 and 3, the mode of failure should be the same if the bar is isolated or is adjacent to another bar as in the case of a splice. It seems, therefore, that the empirical equation for splice strength should be applicable to development lengths as well as splices. To check this, Eq. (6) was used to predict strength in tests on development lengths of deformed bars conducted by Ferguson and Thompson[12,13] and Chamberlin.[14] Details of these test specimens are shown in Figs. 12 and 13. The ratios u_t/u_{cal} in Tables 5 and 6 show that Eq. (6) gives values comparable with those for

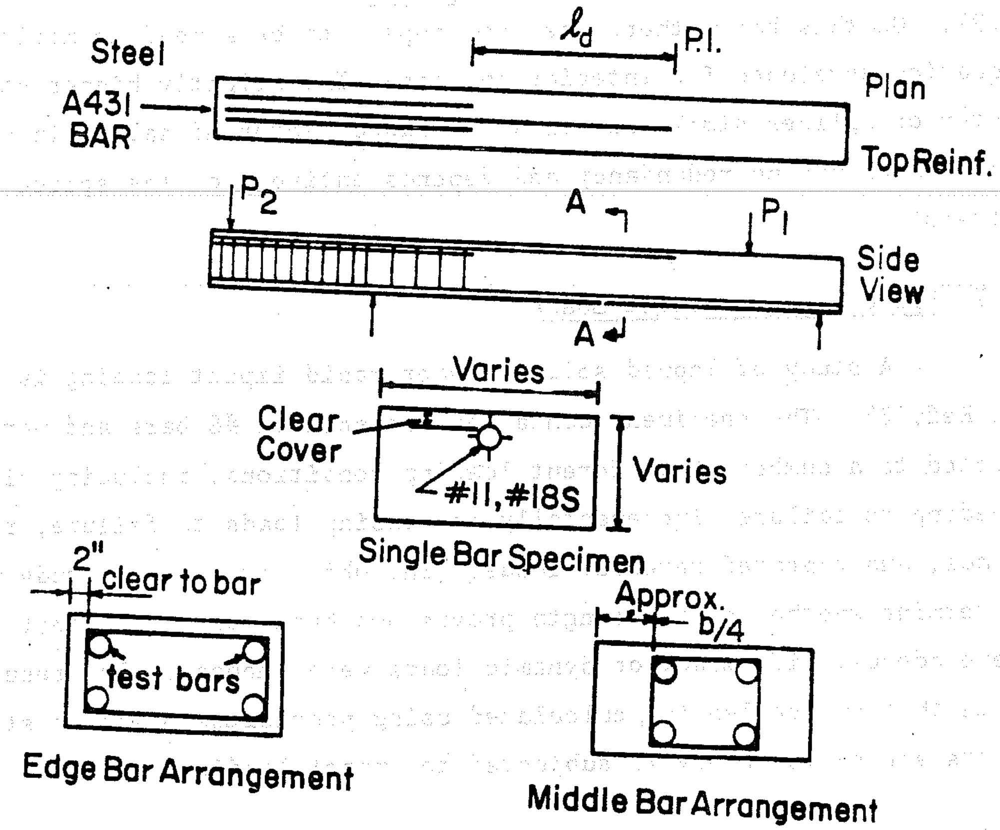


Fig. 12. Details of development length test beams, Ferguson and Thompson (12, 13).

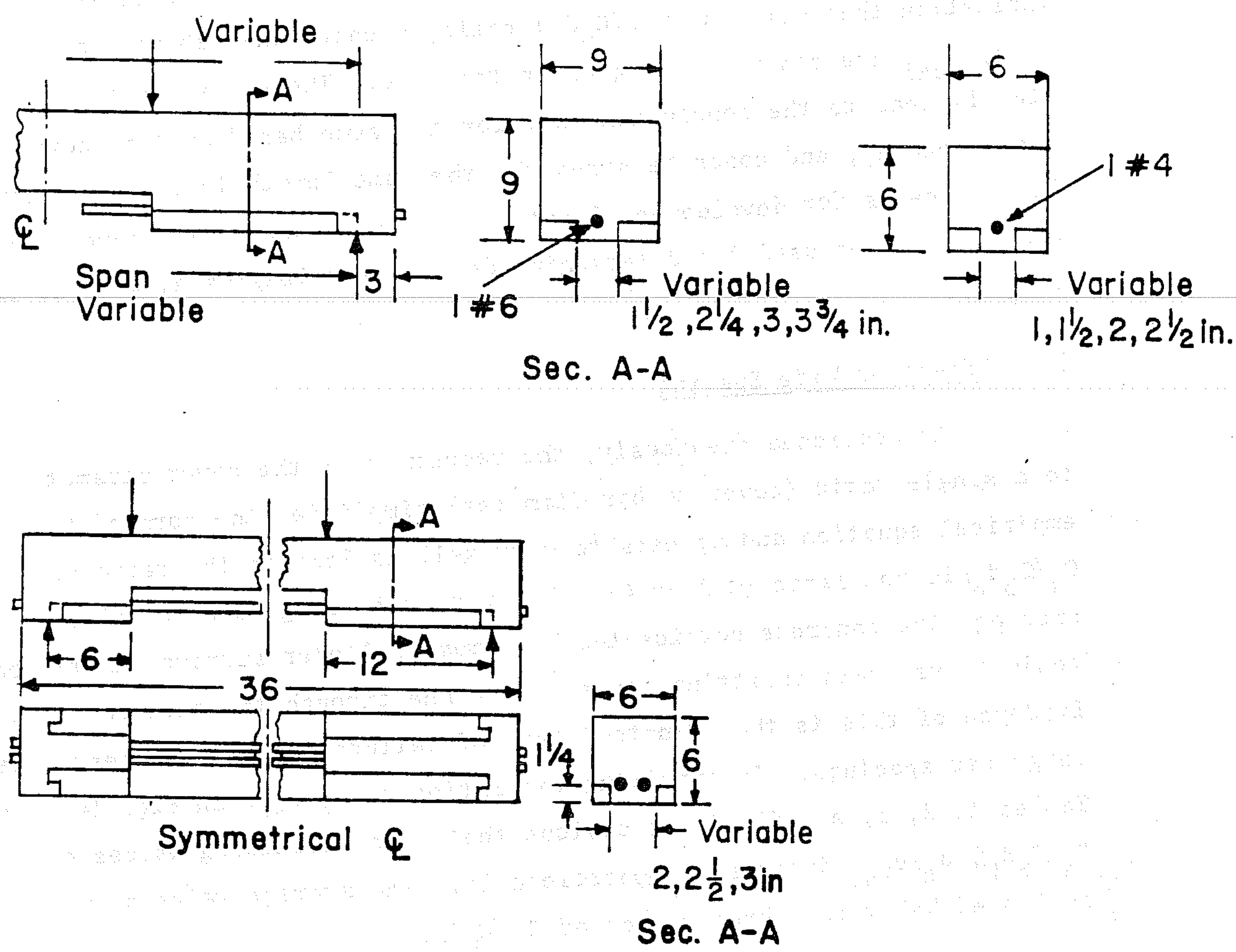


Fig. 13. Details of development length test beams, Chamberlin (14).

splices. Figure 14 is a plot of u_t/u_{cal} versus the ratio $C_s/(C_b d_b)$. The ratio $C_s/(C_b d_b)$ was selected to reflect the restraining influence of large side covers (C_s). As can be seen, there is no definitive trend for splice or development length tests to be segregated. However, there is a definite indication that with the $C_s/(C_b d_b)$ ratio, greater than about 3 or 4, values of u_t/u_{cal} are consistently greater than 1.0. These results plotted in Fig. 14 lead to the conclusion that for the same bar diameter, cover, clear spacing, and concrete strength, the same length is required for a lap splice as for development length. As a result, the same basic equation can be used for determining development lengths as well as lap lengths.

3.9 Effect of Wide Spacing

As mentioned previously, the reduction of the cover parameter to a single ratio (cover to bar diameter) simplifies the form of the empirical equation and appears to work well as long as the ratio of $C_s/(C_b d_b)$ is not large (< 3 or 4). However, with large side or clear spacing, the concrete outside the "minimum" cylinder surrounding the bar tends to restrain splitting across the plane through the anchored bars. Evidence of this is the "V-notch" type of failure observed in tests with large bar spacings. In examining the ratios of u_t/u_{cal} in Fig. 14 (from Tables 1, 2, 5, and 6), it is obvious that with increasing values of $C_s/(C_b d_b)$, u_t/u_{cal} increases proportionally. The average value of u_t/u_{cal} is listed below for three ranges of $C_s/(C_b d_b)$.

$C_s/(C_b d_b)$	$(u_t/u_{cal})_{Avg}$	Standard Deviation
< 3	1.06	0.13
$> 3 < 6$	1.21	0.14
> 6	1.64	0.21

For design purposes it may be sufficient to use a reduction factor on required splice and development lengths in those cases where $C_s/(C_b d_b)$ is greater than 3. It should be noted that crack control provisions may determine maximum spacings of bars in many cases.

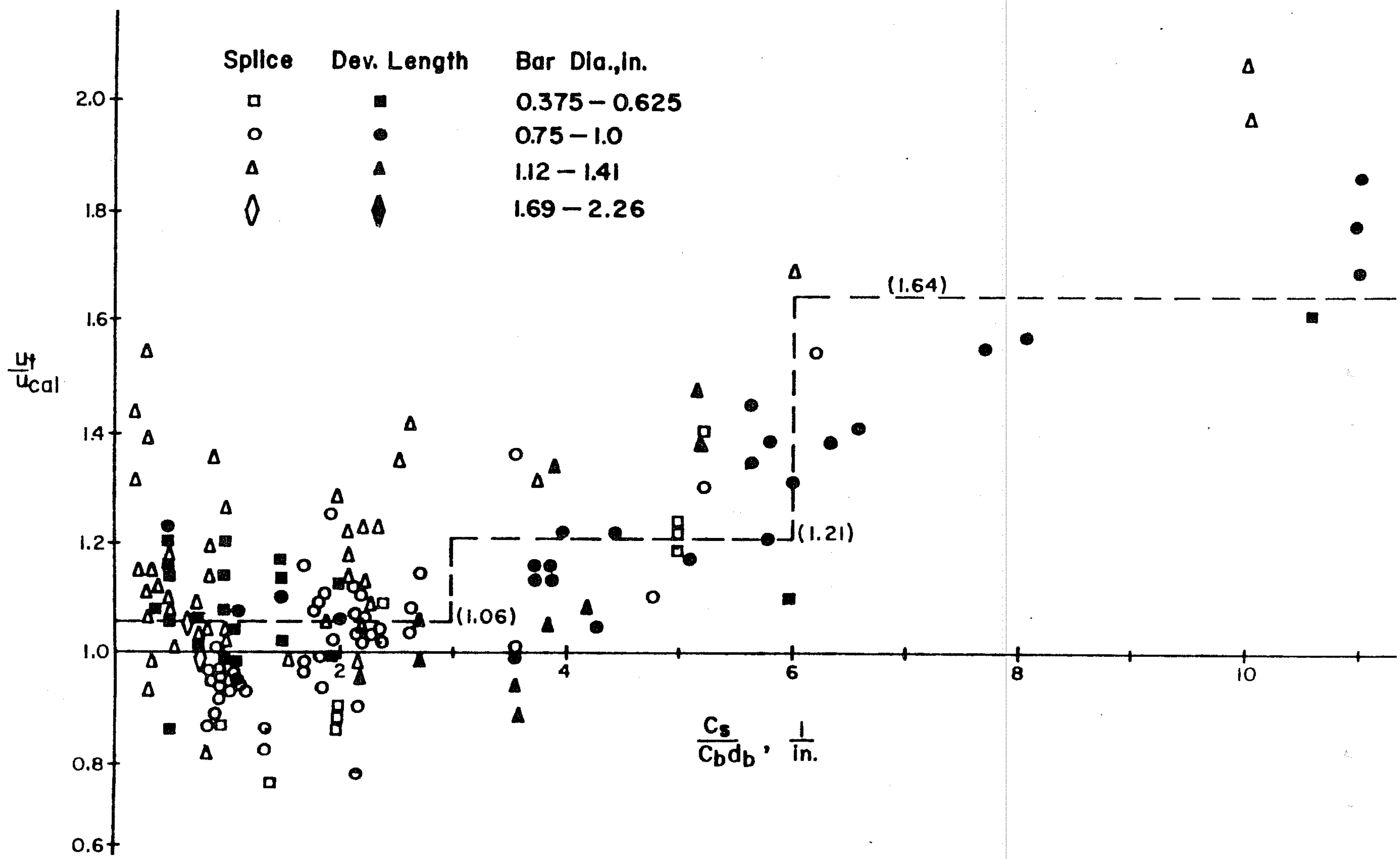


Fig 14. Effect of wide spacing.

4. BARS WITH TRANSVERSE REINFORCEMENT

4.1 Influence of Transverse Reinforcement

The provision of transverse reinforcement adds to the tensile capacity of the plane resisting splitting and increases the overall splice strength. Splitting may occur in splices with transverse reinforcement, but the reinforcement restrains splitting and reduces the tendency for sudden, brittle failures.

The overall strength of a splice with transverse reinforcement can be regarded as the strength of a plain splice together with the strength contributed by the transverse steel, i.e.,

$$u = u_c + u_{tr}$$

u_c can be calculated from Eq. (6). The strength contributed by the transverse steel u_{tr} has been shown by Tepfers[6] to depend on the splice length and amount of transverse steel. The tensile capacity of the transverse reinforcement depends on its yield strength, f_{yt}. In order to evaluate the effect of transverse reinforcement, the results of splice tests (Fig. 15) reported in Refs. 1, 2, 4, and 11, and development length tests reported in Refs. 12 and 16 have been considered. Only tests in which failure occurred before the bars yielded are included. The variations of $u_{tr}/\sqrt{f'_c}$ with several parameters reflecting the confinement provided by the transverse steel were examined. The area of transverse reinforcement A_{tr} was defined as shown in Fig. 16. The spacing s is the average spacing of ties along the development length or splice length. The parameter selected was $A_{tr}f_{yt}/sd_b$. Since $A_{tr}f_{yt}$ represents the force which can be developed at a tie location, it is to be expected that the effectiveness of a tie is inversely proportional to the spacing of the ties and diameter of the bar enclosed. As will be seen later, the parameter is of a form which allows considerable simplification for design purposes.

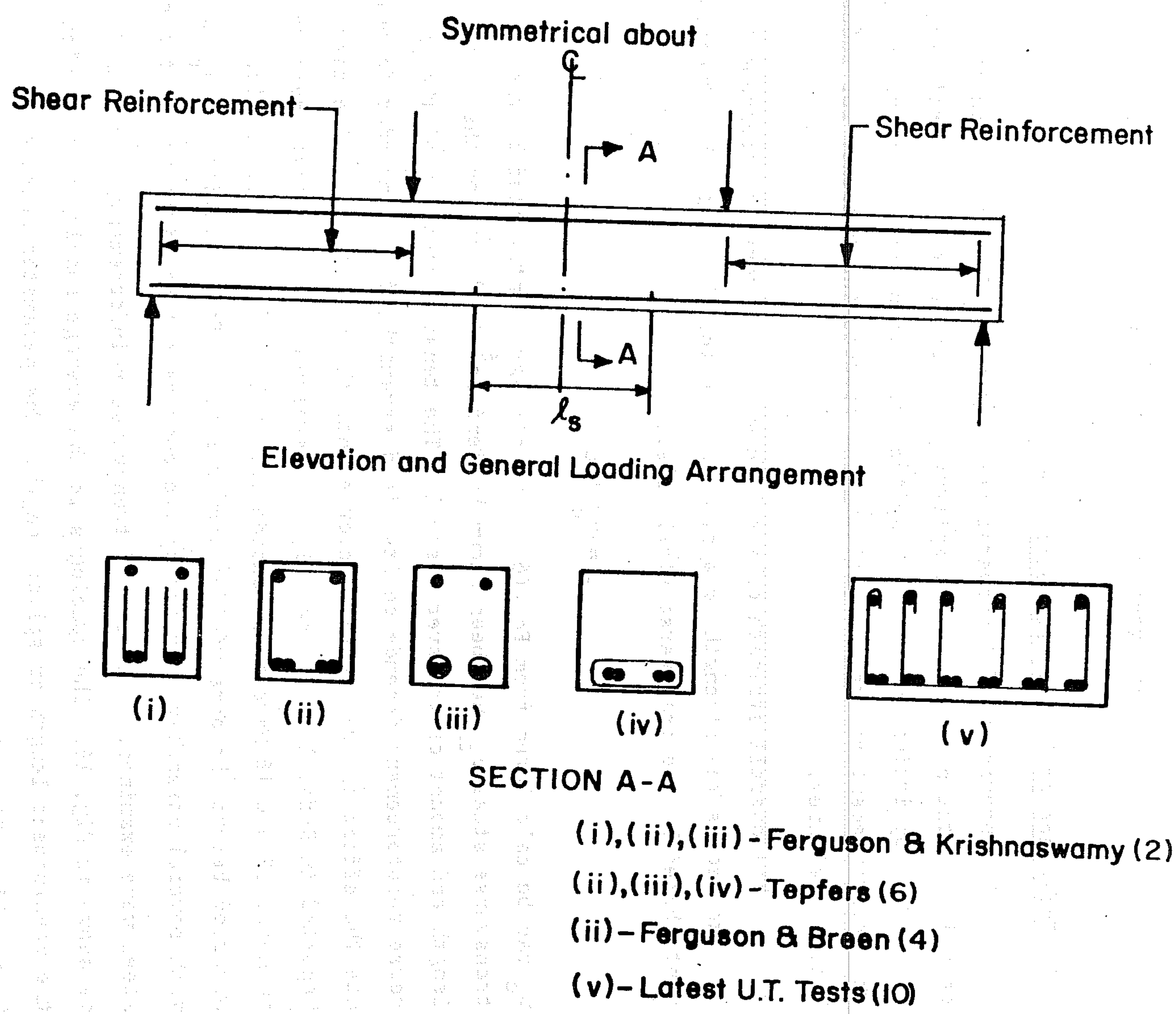
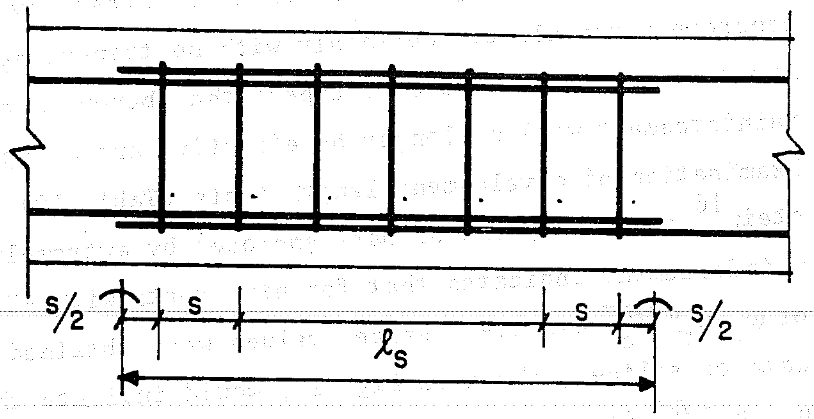


Fig. 15. Details of splice tests with transverse reinforcement.



If spacing is uneven $s = \ell_s$/no. of transverse ties.

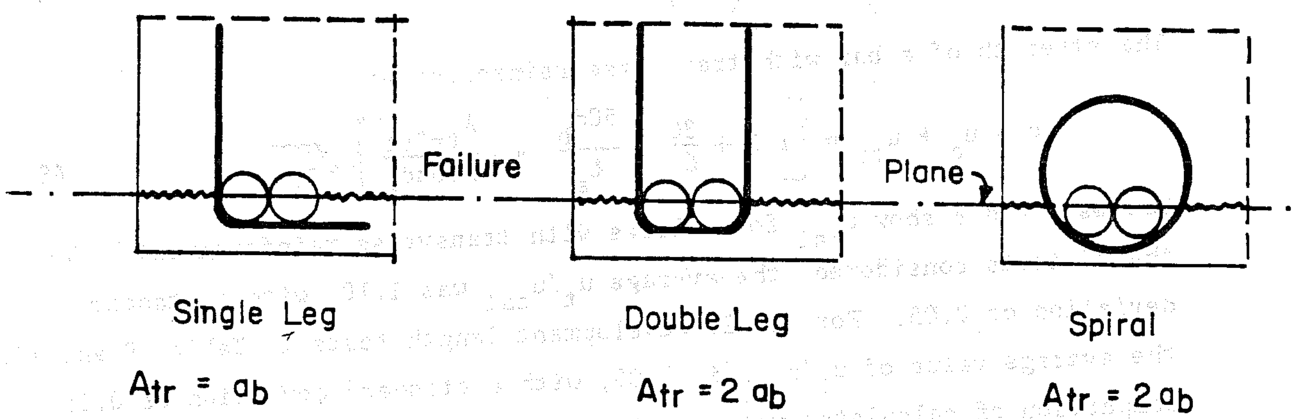


Fig. 16. Transverse reinforcement: definitions.

Using the test results tabulated in Tables 7-10, the value of $(u_t - u_c)/\sqrt{f'_c}$ was calculated and plotted against $A_{tr} f_{yt}/sd_b$ in Fig. 17. As expected, the greater the transverse restraint relative to bar diameter, the greater the strength or increment of stress over that provided by the concrete cover alone. Certainly with no transverse reinforcement, $u_{tr} = 0$. However, it is reasonable to expect that beyond a certain point transverse reinforcement will no longer be effective and an upper limit is needed. Examination of development length tests (Table 10) reported by Mathey and Watstein[16] on development of bars enclosed by extremely heavy transverse reinforcement indicates that for nine tests with #8 bars, the average value of $(u_c - u_t)/\sqrt{f'_c}$ was 2.9. Larger values were obtained with #4 bars. Other data on splices, shown in Fig. 17, would indicate that an upper limit of $u_{tr} = 3\sqrt{f'_c}$ is reasonable. Fitting a straight line through the test results led to the following equation

$$\frac{u_{tr}}{\sqrt{f'_c}} = \frac{1}{500}\left(\frac{A_{tr} f_{yt}}{sd_b}\right) \leq 3 \qquad (7)$$

The strength of a bar with transverse reinforcement is

$$u = u_c + u_{tr} = \left[1.2 + \frac{3C}{d_b} + \frac{50d_b}{\ell_s} + \frac{A_{tr} f_{yt}}{500sd_b}\right]\sqrt{f'_c} \qquad (8)$$

Tables 7 and 8 show u_{cal} for splices with transverse reinforcement. For the 27 tests considered, the average u_t/u_{cal} was 1.10, with a standard deviation of 0.05. For the 27 development length tests in Tables 9 and 10, the average value of u_t/u_{cal} is 1.03, with a standard deviation of 0.15. Comparison of calculated values using Eq. (8) with measured values indicates generally excellent agreement. While it would appear that some of the data varies considerably from the curve shown in Fig. 17, it should be remembered that u_{tr} is an increment added to the strength contributed by the concrete surrounding the bar and thus the differences are not significant.

4.2 Other Tests--Effect of Transverse Reinforcement

A large number of tests have been conducted by researchers in Europe on the strength of bars confined by transverse reinforcement.

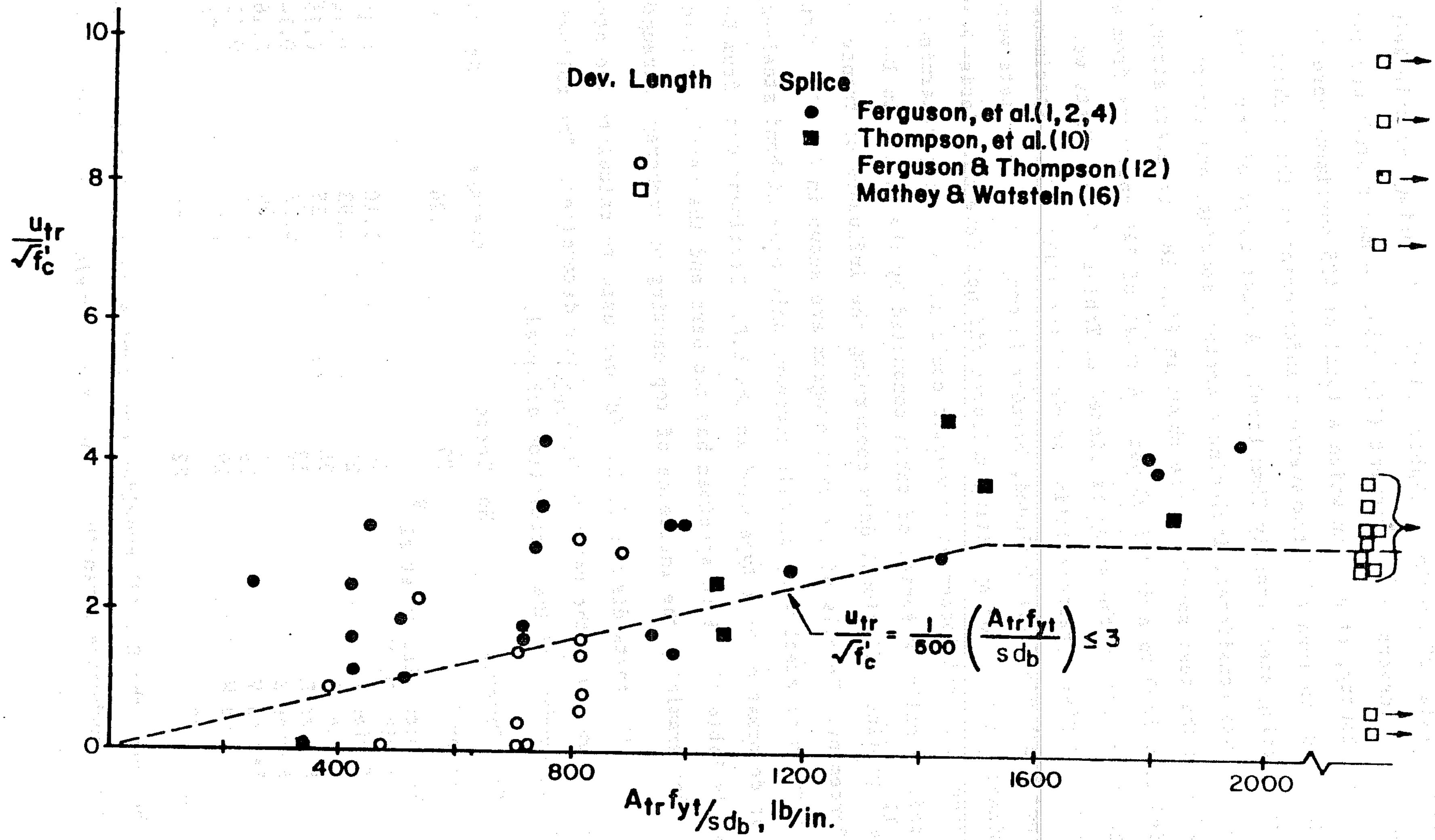


Fig 17. Effect of transverse reinforcement.

Tepfers[6] tested 29 specimens (Table 11) with the prime variable being the amount of transverse reinforcement (Fig. 15). A major study was conducted by Robinson, Zsutty, et al.[9] in which a total of 425 specimens were tested to evaluate the influence of transverse reinforcement on the anchorage capacity of reinforcing steel (mostly 25mm bars). A wide range of transverse steel variables was considered, including diameter, spacing, and strength. Details of the test specimens are shown in Fig. 18. Concrete strength varied from 1200 to almost 6000 psi. A total of 146 specimens from eight different series in the study is listed in Tables 12-16. Tests were selected to give a representative sample of the study. Only specimens which did not reach yield are included, because in many cases the tests were terminated at yield or splitting failures did not develop. Series in which the transverse reinforcement parameter could not be easily determined were omitted. Finally, a series of tests conducted by the C.U.R. in The Netherlands[11] provides additional data concerning the influence of transverse reinforcement. Details of the test program are shown in Fig. 19. Four different types of steel were tested; however, only one--Hi-bond steel--appeared to have deformation of a type used in the U.S. Pertinent data from Ref. 11 are listed in Table 17. Each specimen had two bars and the results provide data useful for examining the influence of top casting on anchorage strength.

For the tests discussed, Eq. (8) was used to calculate the strength of the specimens and the ratio of u_t/u_{cal} was determined. The following is a brief summary of the correlation achieved.

Test Program	No. Tests	Average	St. Dev.	
Tepfers[6]	29	1.24	0.20	
Robinson, Zsutty et al.[9]				
Series D, Y	19	1.10	0.12	
Series B	21	0.93	0.14	
Series A	38	1.25	0.15	106 Tests
Series R	13	0.98	0.14	Avg = 1.13
Series S	7	0.90	0.16	S.D = 0.21
Series V	19	1.02	0.11	
Series W	29	1.14	0.26	
C.U.R.[11]	22	1.08	0.11	

As can be seen, the Eq. (8) provides excellent agreement between calculated and measured anchorage strengths. The lower correlation for Series B of

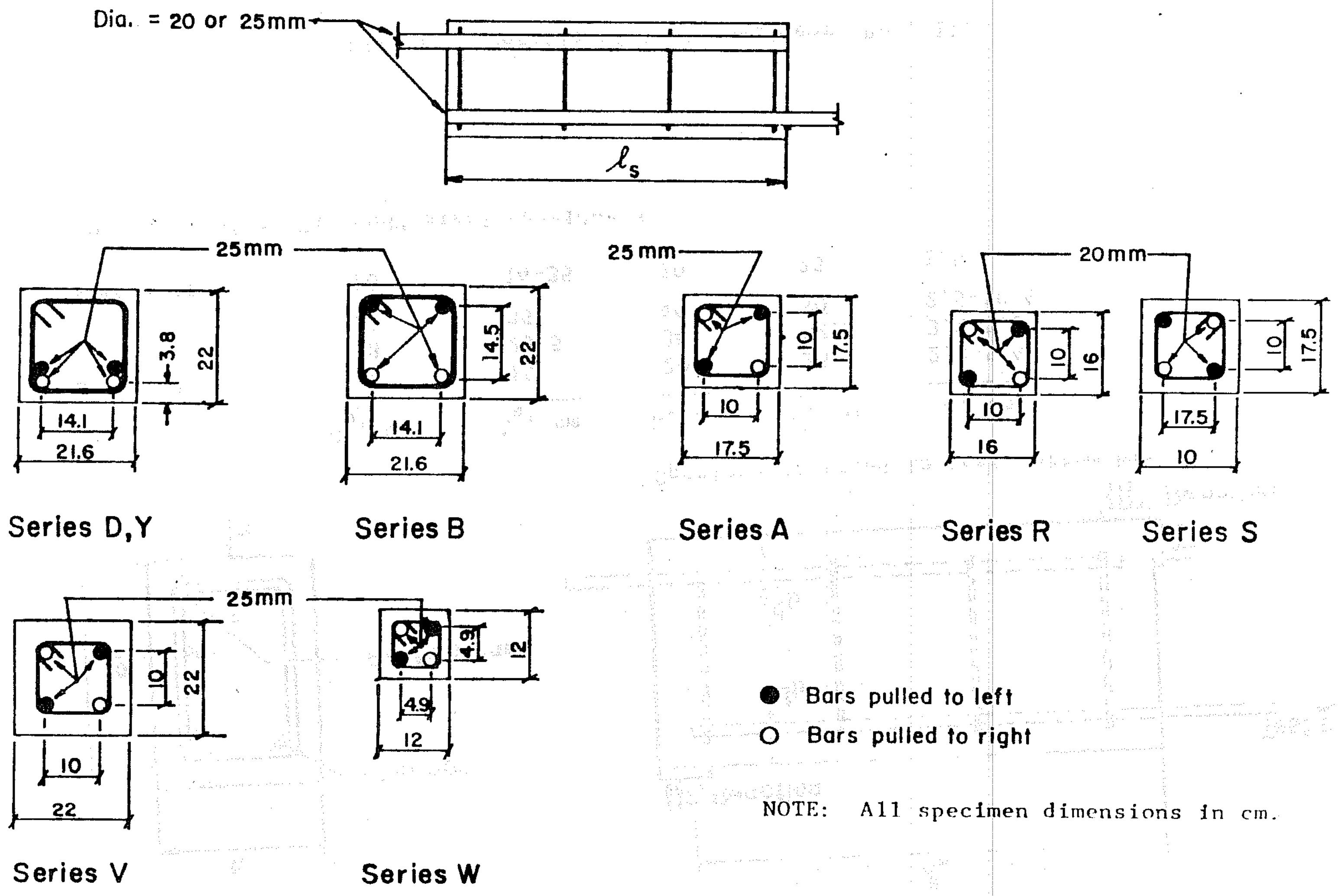
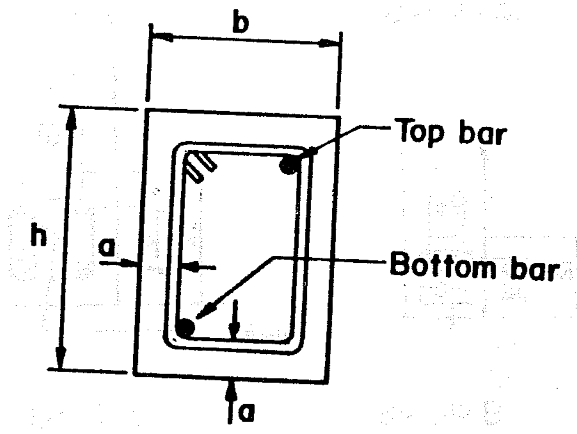
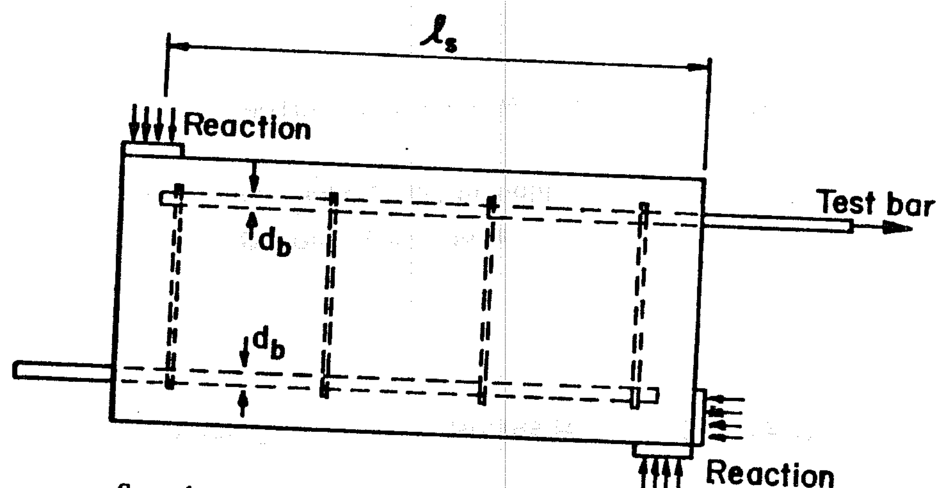


Fig. 18. Details of Tests - Robinson, Zsutty, et al. (9)





Specimen reversed to test bottom bar

	d_b, mm	ℓ_s, cm	b, cm	h, cm	a, cm
Series I	10	14	20	25	2.3-5.4
	18	26.5	34	42.3	3.2-9.0
	26	35	40	50	2.5-10.4
Series II	10	14-35	20	25	1.5

Only test with "Hi-bond" steel considered

Fig. 19. Details of test specimens, Ref. 11.

Ref. 9 may be traced to two factors. Figure 17 shows details of the tests reported in Ref. 9. As the above table indicates, there was excellent agreement with the results of Series D and Y when the bars were in contact. However, when the bars were spread apart as in Series B, there may have been significant shear between the top and bottom bars. Note that when the diagonal bars were stressed in opposite directions, the correlation with predicted stresses was excellent. In these cases, shear between bars is transferred in both direction and may not be as severe as in Series B. Although the average for Series S is low, the sample is small (7 tests) and may not be significant.

4.3 Effect of Top Casting

A major parameter influencing the strength of anchored bars is the position of the bar relative to height of the concrete lift during casting. Current ACI and AASHTO specifications define a top cast bar as one in which 12 in. or more of concrete is cast below the bar. For such bars an increase in development or splice length is required. A limited number of tests in which top cast bars were considered is available. Ferguson and Thompson[12,13] and Thompson, et al.[10] tested a total of 12 specimens with top cast bars (> 12 in. of concrete below the bar). For the 12 tests the average ratio of u_t/u_{cal} [Eq. (8)] is 0.88 with a standard deviation of 0.07. Table 17 lists the results of tests reported in Ref. 11 in which each specimen had both top and bottom bars and the strengths are compared in the last column. It should be noted that the specimens with 10mm bars had about 8 in. of concrete cast below the bar. For these tests, the average u_{top}/u_{bottom} was 0.82 with a standard deviation of 0.12. It is apparent that additional research is needed to evaluate accurately the influence of top casting; however, a decrease of strength of at least 25 to 30 percent for top cast bars is required.

4.4 Lightweight Aggregate Concrete

The present analysis was developed entirely from tests on normal weight or "hard rock" concrete. A modifying factor may be necessary to take into account the difference in the relationship between the tensile strength and the compressive strength of normal and lightweight aggregate concretes. The tensile strength of lightweight aggregate concrete is affected by the moisture conditions at test[20] and any modification that may be required for lightweight aggregate concrete may have to be determined on this basis from tests. Pending such tests, the use of the modifying factors for lightweight concrete contained in current ACI and AASHTO specifications should be continued.

5. PROPOSED DESIGN RECOMMENDATIONS

5.1 Modification of Empirical Equation for Design

Based on the test results analyzed, Eq. (8) represented accurately the strength of an anchored bar in terms of the average bond stress along the bar. For design purposes it is necessary to determine the splice or development length rather than average bond stress. Since $u = f_s d_b / 4\ell_d$,

$$\frac{u}{\sqrt{f'_c}} = \frac{f_s d_b}{4\ell_d \sqrt{f'_c}} = 1.2 + 3\frac{C}{d_b} + \frac{50 d_b}{\ell_d} + \frac{A_{tr} f_{yt}}{500 s d_b}$$

and solving for ℓ_d

$$\ell_d = \frac{d_b\left(\frac{f_s}{4\sqrt{f'_c}} - 50\right)}{\left(1.2 + 3\frac{C}{d_b} + \frac{A_{tr} f_{yt}}{500 s d_b}\right)} \quad (9)$$

Equation (9) expresses the development length (or splice length) in terms of the stress in the bar at the critical section, the bar diameter, concrete strength, cover to diameter ratio, and transverse reinforcement.

Equation (9) can be further simplified in the following manner. The term $(f_s/4\sqrt{f'_c} - 50)$ can be rewritten as $(f_s - 200\sqrt{f'_c})/4\sqrt{f'_c}$. Since $f_s - 200\sqrt{f'_c}$ will be fairly insensitive to the concrete strength, it can be conservatively assumed that $(f_s - 200\sqrt{f'_c})$ equals f_s - 11000 psi ($f'_c \approx$ 3000 psi). Equation (9) becomes

$$\ell_d = \frac{d_b(f_s - 11000)}{4\sqrt{f'_c}\left(1.2 + 3\frac{C}{d_b} + \frac{A_{tr} f_{yt}}{500 s d_b}\right)} \quad (10)$$

For Grade 60 reinforcement and eliminating constants in the denominator

$$\ell_{d60} = \frac{d_b(49000)}{4.8\sqrt{f'_c}\,(1 + 2.5\frac{C}{d_b} + \frac{A_{tr}f_{yt}}{600sd_b})} = \frac{10200d_b}{\sqrt{f'_c}(1 + 2.5\frac{C}{d_b} + K_{tr})} \qquad (11)$$

where $K_{tr} = \frac{A_{tr}f_{yt}}{600sd_b} \leq 2.5$.

For Grade 40 the constant in the numerator is 6040 and for Grade 75 it is 13,300.

The current ACI and AASHTO provisions are based on substituting $1.25f_y$ for f_s in the design equations. Such a substitution can be considered analogous to using a capacity reduction factor of $\varphi = 0.8$, although this is not stated in Commentaries to the ACI and AASHTO specifications. Rather it is assumed that by using a stress 25 percent greater than yield, ductility requirements will be satisfied. It should be noted that in the current provisions [Eq. (1)], the development length is directly proportional to f_s. Therefore, an increase requiring $1.25f_y$ led to a 25 percent increase in development length over that required to develop yield. Examination of Eq. (9) shows that a 25 percent increase in f_s will lead to a somewhat smaller increase in ℓ_d. Therefore, it is recommended that a capacity reduction factor φ be used in development length calculations. Such a factor is used in all other strength calculations in the codes and would provide consistency. The capacity reduction factor is intended to account for deviations in material properties, dimensional errors, and, to some extent, the uncertainty involved in the calculation. There is no rational reason to exclude development length computations from this approach. Based on the data analyzed, a capacity reduction factor $\varphi = 0.8$ seems reasonable.

5.2 Design Recommendations for Development Length and Splice Length of Deformed Bars in Tension

The development length ℓ_d in inches of deformed bars in tension shall be computed as the product of the basic development length of (a) and the applicable modification factor or factors in (b), but ℓ_d shall be not less than 12 in.

(a) The basic development length for Grade 60 reinforcement is

$$\frac{10200 d_b}{\sqrt{f'_c}\,(1 + 2.5\frac{C}{d_b} + K_{tr})\varphi}$$

The capacity reduction factor φ shall be taken as 0.8; C shall be taken as the lesser of the clear cover over the bar or bars or half the clear spacing between adjacent bars and A_{tr} is normal to C; C/d_b shall not be taken as more than 2.5 and the transverse reinforcement term,

$$K_{tr} = \frac{A_{tr} f_{yt}}{600 s d_b} \leq 2.5$$

(b) The basic development length shall be multiplied by the applicable factor or factors for

Grade 40 reinforcement	0.6
Grade 75 reinforcement	1.3
Top reinforcement (from 12 in. to 15 in. of concrete below)	1.3
Wide spacing such that $3 \leq C_s/(C_b d_b) \leq 6$	0.9
Wide spacing such that $C_s/(C_b d_b)$ is greater than 6	0.7
Reinforcement in a flexural member in excess of that required	$(A_s \text{required})/(A_s \text{provided})$

The length of a tension lap splice ℓ_s shall be computed as for development length ℓ_d with the appropriate cover C determined from a consideration of the clear cover and the clear spacing between the splices.

If alternate splices are staggered within a required splice length ℓ_s and the overlap is at least $0.5\ell_s$, the value of clear spacing at a critical section through the end of the splice may be taken without considering the continuous adjacent bars. For lap splices of #14 and #18 bars, minimum transverse reinforcement shall be provided such that $A_{tr} f_{yt}/s d_b \geq 600$ psi.

5.3 Comments and Comparison of Proposed Recommendations with ACI 318-71 and 1974 AASHTO Interim Specifications

The proposed design equation represents a considerable advance over current methods because it takes into account the effect of clear cover, spacing, and transverse reinforcement. By using the same equation for both splice and development lengths, the number of different design conditions is reduced substantially.

The development lengths given in current ACI and AASHTO specifications are compared with the proposed development lengths in Fig. 20 for a Grade 60 steel in 3000 psi and 4500 psi concrete. The proposed development lengths and those given by current specifications are approximately equal for minimum clear covers of about 2 to 2-1/2 in. on sides or bottom for #8, #11, and #14 bars, and at about 3-1/2 in. for #18 bars. Below these values of clear cover, current provisions would tend to overestimate the strength of bars for a given development length and underestimate strength values for cover greater than stated above.

Development lengths proposed for bars with 1-1/2 in. cover which are typical in many structural applications will be greater than those called for in current specifications. For example, a #8 bar with 1-1/2 in. cover (f'_c = 3000 psi) requires a development length of about 34 in. currently and under the proposed design this would be increased to 49 in. Figure 21 shows a comparison of required lengths for Grade 60 steel with f'_c = 3000 psi. Note that for current provisions ℓ_d remains the same regardless of cover or transverse reinforcement. With increase in cover to 3 in. or addition of transverse reinforcement, the required length for #8 and smaller bars is about the same as currently specified. However, for bars larger than #8, the required length is reduced over current specifications if the cover is increased or the transverse steel is added. For example, a #11 bar with 3 in. cover currently requires a development length of about 69 in. This would be reduced to 52 in. under the proposed provisions. Advantage may also be taken of wide spacing which may further reduce the development length required. For slabs or walls with 3/4 in. cover, the development or splice length would be increased over current specifications.

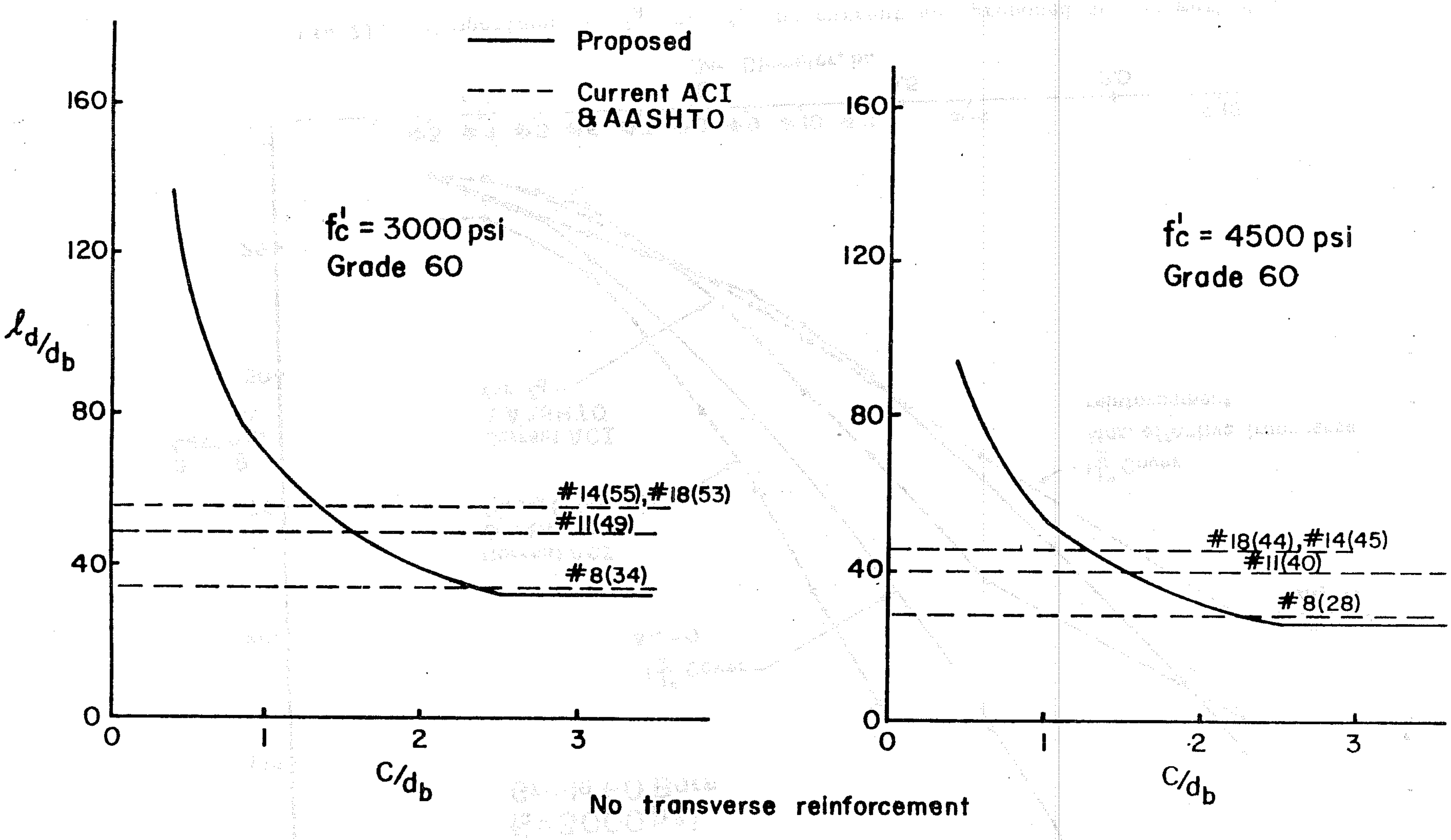


Fig. 20. Comparison of proposed design with current ACI and AASHTO specifications for development length.

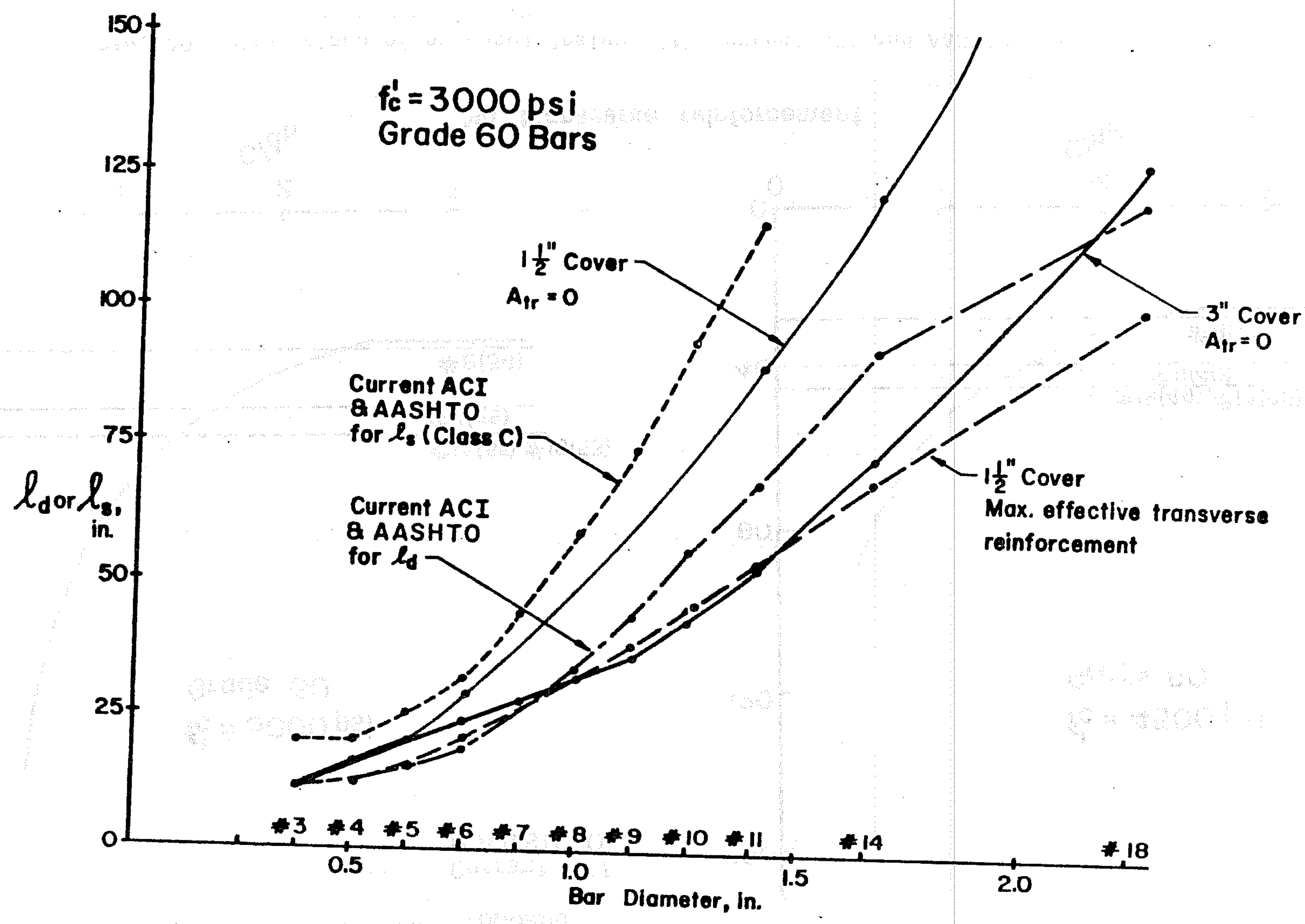


Fig 21. Comparison of ℓ_d or ℓ_s by current and proposed design methods.

With 3/4 in. cover on #5 bars at a clear spacing of 6 in. ($f_c' = 3000$ psi), the proposed development length is about 25 in. and under current provisions is only 14 in.

The reasons for the differences discussed above may be traced to the data on which current provisions are based. The equation for determining development lengths was based largely on tests of large bars by Ferguson and Thompson,[12] and by Mathey and Watstein.[16] Ferguson and Thompson tested single bars in wide beams. The bond beams tested by Mathey and Watstein had extremely heavy transverse reinforcement over the development length. Consequently, higher average bond stresses were obtained which led to shorter development lengths.

The design proposals are also compared with current provisions in Fig. 21 for Class C splices--splices with all the bars lap-spliced in a region of maximum moment and spaced closer than 6 in. on centers--which is the most severe splicing condition. It is seen from Fig. 21 that ACI and AASHTO provisions require a greater splice length than proposed for all bar sizes ($f_y = 60$ ksi, $f_c' = 3000$ psi). Currently lap splices for #14 and #18 bars are prohibited. For a clear cover of 1-1/2 in. on sides or bottom, the proposed provisions represent a reduction in lap lengths from 27 to 24 in. for #6, 59 to 49 in. for #8, and 116 to 90 in. for #11 bars. With larger clear cover and with transverse reinforcement the reductions are even more pronounced. If the maximum effective transverse steel is provided, the lap lengths will be reduced from 27 to 21 in. for #6, from 59 to 33 in. for #8, and from 116 to 54 in. for #11. On the basis of the data considered, there does not appear to be sufficient reason to prohibit lap splices in #14 and #18 bars. However, the splice lengths will be very large unless transverse steel is provided or the cover is increased. Therefore, the proposed provisions suggest lap splices for large bars only if some amount of transverse steel is provided.

6. SUMMARY AND CONCLUSIONS

The basic design equation developed in this study has been well established through successful application to tests from various sources to justify its inclusion in structural design specifications. It represents an improvement on the current ACI and AASHTO provisions. The development and splice lengths were found to be identical and could be expressed in terms of steel stress, concrete strength, bar diameter, minimum side or bottom cover, and transverse reinforcement--factors which have been shown by tests to affect the strength of anchored bars.

Comparison of current provisions for development length with the proposed design recommendations shows that for minimum cover current provisions are unconservative. However, with increase in cover or addition of transverse reinforcement considerable reduction in development length can be realized by using the proposed provisions.

For lap splices in a region of high stress, the proposed provisions lead to considerably shorter splice lengths over those now used. Lap splices for #14 and #18 bars need not be prohibited as far as strength is concerned. Provision of transverse reinforcement is specified for these bar sizes for increased toughness and reduced lap lengths.

B I B L I O G R A P H Y

1. Ferguson, P. M., and Briceno, E.A., "Tensile Lap Splices - Part 1: Retaining Wall Type, Varying Moment Zone," Research Report 113-2, Center for Highway Research, The University of Texas at Austin, July 1969.

2. Ferguson, P. M., and Krishnaswamy, C. N., "Tensile Lap Splices - Part 2: Design Recommendations for Retaining Wall Splices and Large Bar Splices," Research Report 113-3, Center for Highway Research, The University of Texas at Austin, April 1971.

3. Chinn, J., Ferguson, P. M., and Thompson, J. N., "Lapped Splices in Reinforced Concrete Beams," Journal of the American Concrete Institute, Proc. V. 52, October 1955.

4. Ferguson, P. M., and Breen, J. E., "Lapped Splices for High Strength Reinforcing Bars," Journal of the American Concrete Institute, Proc. V. 62, September 1965.

5. Chamberlin, S. J., "Spacing of Spliced Bars in Beams," Journal of the American Concrete Institute, Proc. V. 54, February 1958.

6. Tepfers, R., "A Theory of Bond Applied to Overlapped Tensile Reinforcement Splices for Deformed Bars," Publication 73:2, Division of Concrete Structures, Chalmers Teknisica Hogskola (Chalmers University of Technology), Goteborg, Sweden.

7. Ferguson, P. M., Turpin, R. D., and Thompson, J. N., "Minimum Bar Spacing as a Function of Bond and Shear Strength," Journal of the American Concrete Institute, Proc. V. 50, June 1954.

8. Neville, A., "A General Relation for Strengths of Concrete Specimens of Different Shapes and Sizes," Journal of the American Concrete Institute, Proc. V. 63, October 1966.

9. Robinson, J. R., Zsutty, T. C., Guiorgadze, G., Lima, L. J., Jung, H. L., and Villatoux, J. P., "La Couture des Junctions par Adherence," Annales de L'Institute Technique du Batiment et des Travaux Publics, Supplement to No. 318, Paris, June 1974 (with Appendix).

10. Thompson, M. A., Jirsa, J. O., Breen, J. E., and Meinheit, D. F., "The Behavior of Multiple Lap Splices in Wide Sections," Research Report 154-1, Center for Highway Research, The University of Texas at Austin, February 1975.

11. Commissie voor Uitvoering van Research Ingesteld door de Betonvereniging, "Onderzock naar de Samenwerking van geprofileerd staal met beton," Report No. 23, 1963, The Netherlands (Translation No. 112, 1964, Cement and Concrete Association, London, "An Investigation of the Bond of Deformed Steel Bars with Concrete").

12. Ferguson, P. M., and Thompson, J. N., "Development Length of High Strength Reinforcing Bars in Bond," Journal of the American Concrete Institute, Proc. V. 59, July 1962.

13. Ferguson, P. M., and Thompson, J. N.,"Development Length for Large High Strength Reinforcing Bars," Journal of the American Concrete Institute, Proc. V. 62, January 1965.

14. Chamberlin, S. J., "Spacing of Reinforcement in Beams," Journal of the American Concrete Institute, Proc. V. 53, July 1956.

15. Stoekl, S., "Overlapped Splices of Tensile Reinforcement," Translated from Beton und Stahlbetonbau (10), pp. 229-234, 1972.

16. Mathey, R. G., and Watstein, D., "Investigation of Bond in Beam and Pull-out Specimens with High-Yield-Strength Deformed Bars," Journal of the American Concrete Institute, Proc. V. 57, March 1961.

17. ACI Committee 408, "Bond Stress--The State of the Art," Journal of the American Concrete Institute, Proc. V. 63, November 1966.

18. Goto, Y., "Cracks Formed in Concrete Around Deformed Tension Bars," Journal of the American Concrete Institute, Proc. V. 68, April 1971.

19. ACI Committee 318, Building Code Requirements for Reinforced Concrete (ACI 318-71), American Concrete Institute, Detroit, 1971.

20. Evans, R. H., and Orangun, C. O., "The Use in Prestressed Concrete Structures of Lightweight-Aggregate (Lytag) Concrete," Civil Engineering and Public Works Review, London, V. 60, January 1965.

21. AASHTO Subcommittee on Bridges and Structures, Interim Specifications, Bridges, 1974, American Association of State Highway and Transportation Officials, Washington, D.C., 1974.

22. Rezansoff, T., Bufkin, M. P., Jirsa, J. O., and Breen, J. E., "The Performance of Lapped Splices under Rapid Loading," Research Report 154-2, Center for Highway Research, The University of Texas at Austin, February 1975.

APPENDIX

TABLE 1. COMPARISON OF CALCULATED BOND STRESS WITH TEST VALUES--LAP SPLICES WITHOUT TRANSVERSE REINFORCEMENT, CONSTANT MOMENT $K = 1.0$

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t psi	u_{cal} [Eq. (6)] psi	$\frac{u_t}{u_{cal}}$
			Chinn, Ferguson and Thompson (3)					
D5	11	0.75	1.5	2.0	4180	735	686	1.07
D7	11	0.75	1.27	1.06	4450	552	590	0.94
D9	11	0.75	1.44	1.06	4380	569	585	0.97
D10	7	0.75	1.48	1.06	4370	672	714	0.94
D12	16	0.75	1.62	1.13	4530	512	541	0.95
D13	11	0.75	1.44	2.91	4820	827	719	1.14
D14	11	0.75	0.83	1.10	4820	532	550	0.97
D15	11	0.75	0.62	2.88	4290	718	464	1.54•
D17	16	0.75	0.80	1.10	3580	443	403	1.09
D19	16	0.75	1.70	2.91	4230	696	672	1.03
D20	7	0.75	1.42	1.13	4230	690	719	0.96
D21	11	0.75	1.47	2.91	4480	732	702	1.04
D22	7	0.75	0.80	1.10	4480	613	653	0.94
D23	16	0.75	0.78	1.06	4450	440	444	0.99
D24	16	0.75	0.81	2.88	4450	500	453	1.10*
D25	24	0.75	1.53	1.06	5100	438	500	0.88
D26	24	0.75	0.75	1.10	5100	418	411	1.02
D27	11	0.75	1.50	1.10	4550	558	606	0.92
D29	11	0.75	1.39	1.10	7480	737	777	0.95
D30	16	0.75	1.56	1.10	7480	600	685	0.88
D31	5.5	0.375	0.83	1.10	4700	1054	771	1.37*
D32	11	0.75	1.47	2.88	4700	778	719	1.08
D33	20.25	1.41	1.55	2.03	4830	455	554	0.82
D34	12.5	0.75	1.49	1.06	3800	525	520	1.01
D35	24	0.75	1.45	1.06	3800	408	432	0.95
D36	5.5	0.375	0.56	1.10	4410	853	603	1.41*
D38	11	0.75	1.52	1.56	3160	460	601	0.77
D39	11	0.75	1.56	1.10	3160	446	505	0.88
D40	16	0.75	0.75	2.94	5280	616	475	1.30*
			Ferguson and Breen (4)					
8R18a	18	1.0	1.75	3.26	3470	601	543	1.11
8R24a	24	1.0	1.67	3.28	3530	615	492	1.25
8R30a	30	1.0	1.53	3.27	3030	438	410	1.07
8F36a	36	1.0	1.41	3.29	4650	482	465	1.04
8F36b	36	1.0	1.40	3.24	3770	426	417	1.02
8F39a	39	1.0	1.53	3.27	3650	477	427	1.12
8F42a	42	1.0	1.50	3.30	2660	390	355	1.10
8F42b	42	1.0	1.45	3.27	3830	447	417	1.07
8R42a	42	1.0	1.56	3.30	3310	420	407	1.03
8R48a	48	1.0	1.48	3.26	3040	378	368	1.03

•$C_s/(C_b d_b) > 6$. *$3 < C_s/(C_b d_b) < 6$

TABLE 1 (Continued)

	Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t psi	u_{cal} [Eq. (6)] psi	$\frac{u_t}{u_{cal}}$
				Ferguson and Breen (Continued)					
AY3R	8R64a	64	1.0	1.52	3.27	3550	350	390	0.90
	8R80a	80	1.0	1.50	3.25	3740	302	386	0.78
	8F36k	36	1.0	1.38	1.42	3460	368	396	0.93
	11R24a	33	1.41	1.67	4.65	3720	540	426	1.27
	11R30a	41.25	1.41	1.31	4.65	4030	489	363	1.35
	11F36a	49.5	1.41	1.50	4.65	4570	445	396	1.12
	11F36b	49.5	1.41	1.47	4.63	3350	410	336	1.22
	11F42a	57.75	1.41	1.48	4.63	3530	375	334	1.12
73.5	11F48a	66	1.41	1.53	4.64	3140	383	313	1.22
72.1	11F48b	66	1.41	1.58	4.66	3330	375	328	1.14
83.0	11R48a	66	1.41	1.50	4.67	5620	433	413	1.05
70.6	11R48b	66	1.41	2.06	4.68	3100	367	375	0.98
79.6	11F60a	82.5	1.41	1.59	4.62	2610	332	281	1.18
78.7	11F60b	82.5	1.41	1.50	4.63	4090	328	339	0.97
78.6	11R60a	82.5	1.41	1.41	4.63	2690	327	265	1.23
87.5	11R60b	82.5	1.41	1.75	4.62	3460	365	344	1.06
				Chamberlin (5)					
	4a	6	0.5	1.0	2.5	4370	893	751	1.18*
	4b	6	0.5	1.0	2.5	4370	919	751	1.22*
	4c	6	0.5	1.0	2.5	4370	907	751	1.21*
				Ferguson and Krishnaswamy (2)					
	18S12	60	2.25	3.0	4.56	3160	424	398	1.06
	18S15	93	2.25	2.63	4.50	2860	312	316	0.99
	14S1	45	1.69	2.38	3.46	2710	428	380	1.13
	SP40	15	0.625	0.83	1.25	3220	448	412	1.09
						Average (62 Tests) =			1.07

$*3 < C_s/(C_b d_b) < 6$

TABLE 2. COMPARISON OF CALCULATED BOND STRESS WITH TEST VALUES--LAP SPLICES WITHOUT TRANSVERSE REINFORCEMENT, $K < 1.0$

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t psi	k	u_{cal} [Eq. (6)] psi	$\frac{u_t}{u_{cal}}$
					Ferguson and Briceno (1)				
1	85	1.41	2.0	0.86	2800	191	0.78	204	0.94
5	85	1.41	2.0	0.84	3900	251	0.71	238	1.05
7	57.5	1.41	2.0	0.92	2920	274	0.74	237	1.15
9	85	1.41	2.0	0.85	3060	245	0.72	212	1.15
11	85	1.41	2.0	0.89	3200	247	0.80	222	1.11
12	65	1.41	2.0	1.51	4250	387	0.66	358	1.08
13	44	1.41	2.0	2.17	3380	449	0.88	410	1.09
14	33	1.41	2.0	2.84	3050	438	0.97	419	1.04
15	65	1.41	2.0	2.12	3340	390	0.68	378	1.03
16	44	1.41	3.0	2.12	3060	441	0.78	404	1.09
17	50	1.41	2.0	2.86	3550	419	0.81	409	1.02
19+	57.5	1.41	2.0	0.88	3720	365	0.74	262	1.39
20*	85	1.41	2.0	0.87	3250	343	0.65	221	1.55
22	50	1.41	2.0	2.86	3900	543	0.70	428	1.26
27	42.3	1.41	2.0	1.11	3270	333	0.91	298	1.11
28+	44	1.41	2.0	2.48	3290	481	0.87	405	1.19
1a	47	1.00	2.0	1.00	2775	271	0.75	277	0.98
2a	32	1.00	2.0	1.50	3920	461	0.91	455	1.01
3a+	42	1.00	2.0	0.63	3750	378	0.74	262	1.44
4a	42	1.00	2.0	0.56	4350	354	0.72	268	1.31

+One bar continuous
*Staggered splice
omitted in average calculations

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t psi	k	u_{cal} [Eq. (6)] psi	$\frac{u_t}{u_{cal}}$
					Ferguson and Krishnaswamy (2)				
SP32	50	1.41	1.25	10.59	3280	511	0.63	302	1.69+
SP33	55	1.41	0.75	10.59	3360	485	0.69	236	2.06+
SP34	36	1.41	0.75	10.59	3280	534	0.69	272	1.96+
SP35	20	1.41	2.0	10.59	3310	677	0.77	516	1.31*
SP36	24	1.41	2.0	7.34	3440	698	0.76	492	1.41
SP37	45	1.41	2.0	2.54	3260	542	0.70	401	1.35
SP38	40	1.41	2.0	1.41	2970	384	0.76	325	1.18
SP39	45	1.41	2.0	2.09	3120	400	0.76	392	1.02

+$C_s/(C_b d_b) > 6$
*$3 < C_s/(C_b d_b) < 6$

TABLE 3. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES--LAP SPLICES WITHOUT TRANSVERSE REINFORCEMENT

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t (avg) psi	u_t (edge) psi	u_{cal} [Eq. (6)] psi	$\frac{u_{t(avg)}}{u_{cal}}$	$\frac{u_{t(edge)}}{u_{cal}}$
					Thompson, Jirsa, Breen, and Meinheit (10)					
6.12.4/2/2.6/6	12	0.75	2.0	2.0	3730	873	725	752	1.16	0.96
8.18.4/3/2.6/6	18	1.0	3.0	2.0	4710	832	711	685	1.22	1.04
8.18.4/3/25.4/6	18	1.0	3.0	2.0	2920	629	---	539	1.17	---
8.24.4/2/2.6/6	24	1.0	2.0	2.0	3105	557	534	517	1.08	1.03
11.45.4/1/2.6/6	45	1.41	1.0	2.0	3520	348	297	290	1.20	1.02
11.30.4/2/2.6/6	30	1.41	2.0	2.0	2865	463	395	418	1.11	0.95
11.30.4/2/4.6/6	30	1.41	2.0	2.0	3350	518	476	452	1.15	1.05
11.30.4/2/2.7.4/6	30	1.41	2.0	2.0	4420	650	---	519	1.25	----
11.25.6/2/3.5/5	25	1.41	2.0	3.0	3920	564	405	518	1.09	0.78
14.60.4/2/2.5/5	60	1.69	2.0	2.0	2865	314	288	330	0.95	0.87
14.60.4/2/4.5/5	60	1.69	2.0	2.0	3200	378	346	348	1.09	0.99
					Chinn, Ferguson, and Thompson (3)					
D1	11	0.75	0.75	0.94	3880	548		473	1.16	
D2	(10.25) 11 & 9.5	0.75	0.75	0.94	4820	531		545	0.97	
D3	11	0.75	1.50	1.50	4350	608		700	0.87	
D4	16	0.75	1.50	1.50	4470	531		638	0.83	
D6	11	0.75	1.16	1.06	4340	540		582	0.93	
D8	11	0.75	1.48	1.06	4570	587		598	0.98	
					Chamberlin (5)					
3a	6	0.5	1.0	1.0	4450	666		758	0.88	
3b	6	0.5	1.0	1.0	4450	671		758	0.88	
3c	6	0.5	1.0	1.0	4450	681		758	0.90	

TABLE 4. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES--LAP SPLICES WITHOUT TRANSVERSE REINFORCEMENT, TEPFERS (6)

Test	ℓ_s cm	d_b mm	C_b cm	C_s cm	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
123-S1	24	16	2.5	2.4	3250	673	515	1.31
123-S2	40	16	2.5	2.4	4320	621	506	1.23
123-S3	56	16	2.5	2.4	4340	496	469	1.06
123-S4	72	16	2.5	2.4	4170	427	439	0.97
123-S7	96	16	2.5	2.4	4400	369	433	0.85
657-1	52	16	2.0	2.4	3230	426	368	1.16
657-2	72	16	2.0	2.4	3230	374	345	1.09
657-3	102	16	2.0	2.4	3180	321	323	0.99
657-4	132	16	2.0	2.4	3180	267	313	0.85
657-13	72	16	3.2	2.4	3200	437	364	1.20
657-14	72	16	1.0	2.4	3200	349	237	1.47
657-22	6	12	2.0	2.05	3090	1023	900	1.14
657-23	12	12	2.0	2.05	3530	906	665	1.36
657-24	24	12	2.0	2.05	4050	796	553	1.44
657-25	36	12	2.0	2.05	3190	580	443	1.31
657-25A	66	12	2.0	2.05	4150	419	458	0.91
657-37	8	16	2.0	1.65	3390	914	832	1.09
657-38	16	16	2.0	1.65	3540	650	553	1.18
657-39	32	16	2.0	1.65	3370	579	394	1.47
657-40	48	16	2.0	1.65	3900	457	372	1.23
657-40A	88	16	2.0	1.65	3740	324	318	1.02
715-56-52	52	16	0.5	3.25	3920	539	230	2.34
715-56-53	52	16	1.5	3.35	4060	613	354	1.73
716-56-54	52	16	3.5	3.38	3960	613	571	1.07
716-56-55	52	16	5.0	3.4	5120	677	652	1.04
732-1	52	16	1.9	2.45	2440	409	311	1.31
732-2	52	16	2.4	2.45	3310	440	416	1.06
732-3	52	16	1.8	2.45	5060	551	435	1.27
732-4	52	16	2.1	2.43	6570	660	541	1.22
732-5	52	16	1.6	2.45	8120	749	517	1.45
732-6	52	16	1.7	2.45	9095	677	565	1.20
732-7	52	16	2.3	2.43	1300	230	254	0.91
732-9	52	16	2.3	2.43	3055	546	390	1.40
732-10	52	16	2.2	2.43	3920	573	430	1.33
732-11	52	16	2.1	2.43	2270	436	318	1.37
732-12	52	16	2.1	2.40	1100	236	221	1.07
732-13	52	16	2.6	2.40	1410	240	271	0.88
732-14	52	16	2.6	2.425	1860	289	314	0.92
732-15	52	16	2.3	2.45	4050	460	449	1.03
732-16	52	16	2.6	2.475	4675	493	505	0.98
732-17	52	16	2.1	2.475	6620	539	543	0.99

TABLE 4 (Continued)

Test	ℓ_s cm	d_b mm	C_b cm	C_s cm	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
732-28	52	16	2.3	2.42	6200	714	555	1.28
732-30	52	16	2.6	2.40	6270	719	573	1.25
732-35	52	16	1.9	2.48	5290	617	458	1.35
732-36	52	16	1.9	2.45	13300	643	726	0.88
732-37	52	16	1.8	2.52	12540	490	684	0.72
732-42	52	19	3.6	3.10	4880	631	553	1.14
732-43	52	19	3.9	3.075	3220	464	447	1.04
732-44	52	16	5.7	2.45	3150	514	412	1.25
732-45	52	16	4.9	2.45	2780	534	387	1.38
732-46	52	16	0.1	4.80	3880	434	182	2.38
732-47	52	16	1.8	4.80	2570	397	310	1.28
732-48	52	16	1.7	5.85	2880	491	318	1.54
732-49	52	16	0.1	2.40	2400	426	143	2.97
732-50	52	16	7.4	0.95	2700	356	235	1.52
732-51	52	16	1.9	2.48	3730	436	385	1.13
732-52	52	16	1.9	2.42	3550	426	375	1.13
732-53	52	16	2.0	2.48	1620	264	261	1.01
732-54	52	16	1.7	2.52	5700	514	447	1.15
732-55	52	16	1.8	2.52	7490	527	529	1.00
732-58	52	16	0	0	2230	111	129	0.86
732-59	72	19	2.4	2.05	2270	261	274	0.95
732-60	32	19	2.6	2.05	2270	363	352	1.03
732-61	72	19	1.9	2.02	2300	237	264	0.90
732-62	32	19	2.1	2.02	2530	284	370	0.77
732-63	22	12	1.9	2.75	2410	543	426	1.27
732-64	32	12	1.7	2.78	1780	469	309	1.52
732-65	42	12	1.6	2.80	2400	393	324	1.21
732-66	52	12	2.0	2.80	2400	389	360	1.08
732-67	22	12	1.5	2.80	2770	457	404	1.13
732-68	32	12	1.4	2.75	2770	374	346	1.08
732-69	42	12	1.4	2.75	2620	413	314	1.32
732-70	52	12	1.2	2.78	2620	359	293	1.22
732-71	52	16	2.3	4.62	2990	457	385	1.19
732-72	52	16	2.4	5.88	3280	559	415	1.34
732-73	52	16	2.5	7.15	3370	483	431	1.12
732-74	52	16	6.6	2.375	3230	479	409	1.17
732-75	52	16	8.3	2.375	3230	503	409	1.23
732-76	52	16	9.6	2.35	890	144	213	0.68
732-77	52	16	9.5	2.375	2040	450	325	1.39
732-40	32	10	1.8	2.43	3180	569	460	1.24
732-41	32	10	1.5	2.60	3320	689	418	1.65

TABLE 4 (Continued)

Test	ℓ_s cm	d_b mm	C_b cm	C_s cm	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
747-1	52	25	3.7	6.25	3600	471	482	0.98
747-2	72	25	4.0	6.25	3650	511	467	1.09
747-3	92	25	4.0	6.25	3180	397	415	0.96
747-4	52	25	3.7	6.25	2920	519	434	1.19
747-5	92	25	4.9	6.20	3800	554	520	1.06
747-6	132	25	3.6	6.20	4360	451	427	1.06
747-7	52	32	5.1	5.50	3480	486	534	0.91
747-8	92	32	3.8	5.50	2850	386	347	1.11

TABLE 5. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES--DEVELOPMENT LENGTH TESTS, FERGUSON AND THOMPSON (12, 13)

Test	ℓ_d in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
B13	15.75	0.875	1.73	8.53	3800	816	611	1.34*
B19	15.75	0.875	1.69	8.64	3000	743	535	1.39*
B20	15.75	0.875	1.72	8.53	5430	1060	728	1.45*
B46	21	0.875	1.47	8.53	4110	754	533	1.41**
B47	21	0.875	1.62	8.53	2580	588	449	1.31**
B16	21	0.875	0.81	8.56	3910	639	379	1.69**
B27	21	0.875	1.53	8.5	5950	905	657	1.38**
B34	21	0.875	2.59	8.53	2380	674	593	1.14*
B38	21	0.875	2.62	8.53	3720	871	748	1.16*
B6	21	0.875	1.47	5.5	3980	546	525	1.04*
B45	21	0.875	1.50	6.61	3560	587	502	1.17*
B44	28	0.875	1.66	6.5	3060	570	467	1.22*
A1	15	0.375	0.69	2.75	2470	638	396	1.61**
A4	12	0.375	1.25	2.81	2690	730	661	1.10*
B35	28	0.875	2.44	8.53	2980	686	609	1.13*
B36	28	0.875	2.56	8.53	3180	747	650	1.14*
B37	28	0.875	0.78	8.53	2930	521	294	1.77**
B39	28	0.875	2.69	8.44	3340	711	693	1.02*
B40	28	0.875	0.90	8.73	3780	651	360	1.81**
B42	35	0.875	1.66	8.51	2950	535	442	1.21*
B4	35	0.875	0.78	5.56	3360	470	297	1.58**
B3	35	0.875	1.66	5.56	2810	496	431	1.15*
B1	35	0.875	2.09	6.5	3470	561	566	0.99*
B43	35	0.875	0.97	6.53	3590	535	346	1.55**
C1	45	1.41	1.41	8.31	3300	357	331	1.08*
C8	45	1.41	1.56	8.31	3920	399	381	1.05*
C9	45	1.41	2.69	8.31	3020	448	466	0.96
C10	33.8	1.41	1.50	8.22	3050	476	358	1.33*
C11	33.8	1.41	1.56	11.36	3760	566	405	1.39*
C33	33.8	1.41	3.0	11.47	2900	554	520	1.06
C40	49.4	1.41	2.0	10.11	3310	353	395	0.89*
C20	50.75	1.41	1.56	11.42	3600	522	354	1.47*
C35	50.75	1.41	3.0	11.53	3430	521	525	0.99
C38	63.3	1.41	2.0	10.11	3410	361	383	0.94

$*3 < C_s/(C_b d_b) < 6$

$**C_s/(C_b d_b) > 6$

TABLE 6. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES--DEVELOPMENT LENGTH TESTS, CHAMBERLIN (14)

Test	ℓ_d in.	d_b in.	C_b in.	C_s in.	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
Series II	10-2/3	0.50	1	0.25	3680	359	305	1.17
	10-2/3	0.50	1	0.5	3680	429	396	1.08
	10-2/3	0.50	1	0.75	3680	496	488	1.02
	10-2/3	0.50	1	1	3680	573	578	0.99
Series III	6	0.50	1	0.25	4470	486	459	1.06
	6	0.50	1	0.5	4470	674	559	1.20
	6	0.50	1	0.75	4470	751	659	1.14
	6	0.50	1	1	4470	850	760	1.12
	16	0.75	1	0.375	4470	415	337	1.23
	16	0.75	1	0.75	4470	471	437	1.08
	16	0.75	1	1.125	4470	556	504	1.10
	16	0.75	1	1.5	4470	534	504	1.06
	10-2/3	0.50	1	0.25	5870	440	386	1.14
	10-2/3	0.50	1	0.5	5870	492	501	0.98
	6	0.50	1	0.25	5870	633	526	1.20
	6	0.50	1	0.5	5870	730	641	1.14
	6	0.50	1	0.75	5870	878	756	1.16
Series IV	6	0.50	1	0.25	4540	496	463	1.07
	6	0.50	1	0.375	4540	534	513	1.04
	6	0.50	1	0.5	4540	587	563	1.04
	12	0.50	1	0.25	4540	280	322	0.87
	12	0.50	1	0.375	4540	374	372	1.01
	12	0.50	1	0.5	4540	416	423	0.98

TABLE 7. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: SPLICES WITH TRANSVERSE REINFORCEMENT, FERGUSON ET AL. (1, 2, 4)

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	$\frac{A_{tr}f_{yt}}{sd_b}$ psi	f'_c psi	u_t psi	u_c psi	$\frac{u_t - u_c}{\sqrt{f'_c}}$	u_{cal} psi	$\frac{u_t}{u_{cal}}$
					Ferguson and Breen						
8F30b	30	1.0	1.50	4.26	505	2610	473	376	1.9	426	1.11
8F36c	36	1.0	1.47	4.27	420	2740	422	366	1.1	416	1.01
75.2 8F36d	36	1.0	1.53	4.27	715	3580	522	429	1.6	485	1.08
79.5 8F36e	36	1.0	1.47	4.28	420	4170	552	451	1.6	511	1.08
77.8 8F36f	36	1.0	1.50	4.27	715	3780	540	435	1.7	493	1.09
75.2 8F36g	36	1.0	1.53	4.26	420	3070	522	397	2.3	442	1.18
8F36h	36	1.0	1.59	4.26	975	1910	383	321	1.4	406	0.94
8F36j	36	1.0	1.50	4.28	975	1820	440	302	3.2	385	1.14
82.0 11R36a	49.5	1.375	2.02	4.64	735	3020	570	413	2.9	493	1.15
					Ferguson and Briceno						
SP24	57.5	1.41	2.0	0.90	250	3610	398	261	2.3	296	1.34
SP25	42.3	1.41	2.0	0.93	750	3340	531	280	4.3	367	1.45
SP26	42.3	1.41	2.0	1.09	750	3200	483	293	3.4	378	1.28
					Ferguson and Krishnaswamy						
14S2	54	1.69	2.4	3.44	520	3345	466	406	1.0	466	1.00
14S3	30	1.69	2.4	3.41	940	3020	549	455	1.7	558	0.98
18S1	72	2.25	3.0	4.54	450	2710	513	352	3.1	398	1.29
18S4	60	2.25	3.0	4.55	1420	3940	619	444	2.8	622	0.99
18S2	60	2.25	3.0	4.53	1175	2620	493	362	2.6	482	1.02
18S3	72	2.25	3.0	4.53	345	4650	464	461	0	508	0.91
14S4	30	1.69	2.38	3.44	1795	3200	704	466	4.2	635	1.11
14S6	36	1.69	2.38	3.44	1800	3570	704	464	4.0	643	1.09
18S11	60	2.25	3.0	4.56	975	3220	583	401	3.2	512	1.14
18S13	48	2.25	3.0	4.56	1950	3400	696	440	4.4	615	1.13

TABLE 8. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: SPLICES IN WIDE BEAMS WITH TRANSVERSE REINFORCEMENT, THOMPSON et al. (10)

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_c psi	$\frac{u_t - u_c}{\sqrt{f'_c}}$	u_{cal} psi	$\frac{u_t}{u_{cal}}$
8.15.4/2/2.6/6	15	1.0	2.0	2.0	1440	3510	902	624	4.7	794	1.14
11.20.4/2/2.6/6	20	1.41	2.0	2.0	1050	3400	617	524	2.4	646	0.95
11.20.4/2/2.6/6	20	1.41	2.0	2.0	1840	3620	742	540	3.4	720	1.03
11.30.4/2/2.6/6	30	1.41	2.0	2.0	1060	3060	528	431	1.7	548	0.96
11.20.4/2/2.6/6	20	1.41	2.0	2.0	1510	3260	728	512	3.8	683	1.07

TABLE 9. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: DEVELOPMENT LENGTH TESTS WITH TRANSVERSE REINFORCEMENT, FERGUSON AND THOMPSON (12)

Test	ℓ_d in.	d_b in.	C_b in.	C_s in.	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_c psi	$\frac{u_t - u_c}{\sqrt{f'_c}}$	u_{cal} psi	$\frac{u_t}{u_{cal}}$
C14E	33.8	1.41	1.63	5.57	709	3810	442	416	0.4	503	0.88
C18M	33.8	1.41	1.56	5.58	710	3980	505	417	1.4	507	1.00
C15E	33.8	1.41	3.00	5.38	706	2960	480	526	0	603	0.80
C25M	33.8	1.41	3.00	5.39	707	3090	530	537	0	615	0.86
C19M	50.75	1.41	1.63	5.34	815	3430	449	355	1.6	450	1.00
C23M	50.75	1.41	1.50	5.34	806	2970	479	315	3.0	403	1.18
C21M	50.75	1.41	3.06	5.43	810	3120	550	508	0.8	598	0.92
C26M	50.75	1.41	3.00	5.36	810	2730	541	468	1.4	552	0.98
C27M	50.75	1.41	3.00	5.38	810	3240	545	510	0.6	602	0.91
C16E	67.5	1.41	1.50	5.6	515	4090	480	348	2.1	413	1.16
C3E	56.2	1.41	1.81	3.75	379	3530	428	375	0.9	420	1.02
C4E	56.2	1.41	2.19	3.72	879	3620	597	428	2.8	534	1.12
H7	90.0	2.25	4.5	9.98	472	4050	540	537	0	597	0.91

TABLE 10. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: DEVELOPMENT LENGTH TESTS WITH HEAVY TRANSVERSE REINFORCEMENT, MATHEY AND WATSTEIN (16)

Test	ℓ_d in.	d_b in.	C_b in.	C_s in.	$\frac{A_{tr} f_{yt}}{sd_b}$ psi	f'_c psi	u_t psi	u_c psi	$\frac{u_t - u_c}{\sqrt{f'_c}}$	u_{cal} psi	$\frac{u_t}{u_{cal}}$
4-7-1	7	0.5	1.75	3.75	22500	4265	1638	997	9.8	1193	1.37
4-7-2	7	0.5	1.75	3.75	22500	4210	1572	991	9.0	1185	1.33
4-10.5-2	10.5	0.5	1.75	3.75	22500	4055	1361	897	7.3	1088	1.25
4-10.5-3	10.5	0.5	1.75	3.75	22500	3675	1341	853	8.1	1035	1.30
4-14-2	14	0.5	1.75	3.75	22500	3710	892	821	0.4	1003	0.89
8-7-1	7	1.0	1.5	3.5	11240	4005	1023	812	3.3	1000	1.02
8-14-1	14	1.0	1.5	3.5	11240	3585	598	555	0.7	734	0.81
8-14-2	14	1.0	1.5	3.5	11240	4055	760	590	2.7	781	0.97
8-21-1	21	1.0	1.5	3.5	11240	4235	737	525	3.3	720	1.02
8-21-2	21	1.0	1.5	3.5	11240	3495	635	477	2.7	654	0.97
8-28-1	28	1.0	1.5	3.5	11240	4485	691	501	2.8	702	0.98
8-28-2	28	1.0	1.5	3.5	11240	3700	643	455	3.1	637	1.01
8-34-1	34	1.0	1.5	3.5	11240	3745	678	438	3.9	612	1.11
8-34-2	34	1.0	1.5	3.5	11240	3765	661	439	3.6	623	1.06

TABLE 11. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES--SPLICES WITH TRANSVERSE REINFORCEMENT, TEPFERS (6)

Test	ℓ_s cm	d_b mm	C_b cm	C_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
123-S8	24	16	2.5	2.4	1490	3830	827	743	1.11
123-S9	40	16	2.5	2.4	900	4250	694	618	1.12
123-S10	56	16	2.5	2.4	640	4200	631	545	1.16
123-S13	56	16	2.5	2.4	1920	4280	777	662	1.17
123-S14	72	16	2.5	2.4	1490	3970	593	617	0.96
123-S19	72	16	2.5	2.4	1490	3900	521	611	0.85
657-5	32	16	2.0	2.4	1240	3010	660	544	1.21
657-6	52	16	2.0	2.4	755	3010	561	439	1.29
657-7	72	16	2.0	2.4	540	3170	534	402	1.33
657-8	102	16	2.0	2.4	380	3170	460	365	1.26
657-9	52	16	2.0	2.4	290	3440	506	415	1.22
657-10	52	16	2.0	2.4	1400	3440	749	545	1.37
657-12	52	16	2.0	2.4	610	3250	740	541	1.37
657-11	52	16	2.0	2.4	755	3250	553	456	1.21
715-56-4	32	16	2.0	2.4	2610	4015	976	662	1.47
715-56-6	32	16	2.0	2.4	2610	1515	543	407	1.33
715-56-7	32	16	2.0	2.4	2610	6450	1116	840	1.33
715-56-9	52	16	2.0	2.4	2910	3810	710	584	1.21
715-56-10	52	16	2.0	2.4	2610	4120	726	609	1.19
715-56-64	22	12	1.5	2.45	3480	2530	817	537	1.52
715-56-65	32	12	2.3	2.4	3480	2300	751	567	1.32
715-56-71	22	16	2.0	2.48	2610	845	239	337	0.71
715-56-72	32	16	2.0	2.5	2610	2480	523	520	1.00
715-56-73	42	16	1.9	2.48	2610	2670	590	500	1.18
715-56-61	32	16	2.0	2.48	2610	5080	986	745	1.32
747-13	52	32	4.0	5.55	1310	4000	813	673	1.21
747-14	52	32	4.0	5.55	2324	3830	930	682	1.36
747-15	52	32	4.0	5.55	3630	3920	1006	691	1.46
747-12	52	25	4.2	6.25	3630	3960	1243	739	1.68

TABLE 12. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: EFFECT OF TRANSVERSE REINFORCEMENT, ROBINSON, ZSUTTY et al. (9)

Series D, Y: d_b = 25mm; C_b = C_s = 2.5cm

Test	ℓ_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
Series D						
20-12φ5	64.5	630	4680	595	505	1.18
20-9φ6	64.5	720	4730	574	521	1.10
Series Y						
20-12φ5	53	870	2430	505	409	1.24
20-6φ5	53	410	4370	590	487	1.21
20-12φ6	53	1480	2790	642	502	1.27
20-5φ6	53	610	4090	505	499	1.01
20-5φ8	53	780	3930	505	509	0.99
20-4φ10	53	980	3940	573	534	1.07
30-12φ5b	78	560	2230	451	326	1.38
30-6φ5	78	270	4650	504	433	1.16
30-5φ6	78	420	4000	458	420	1.09
30-5φ6c	78	420	2740	343	348	0.99
30-4φ8	78	400	4330	458	434	1.05
30-4φ8c	78	400	2430	343	325	1.05
40-6φ5	103	220	4090	390	374	1.04
40-5φ6	103	320	3800	390	373	1.04
40-5φ6c	103	320	2200	298	284	1.05
40-4φ8	103	300	4160	390	387	1.01
40-4φ8c	103	300	2270	260	286	0.91

TABLE 13. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: EFFECT OF TRANSVERSE REINFORCEMENT, ROBINSON, ZSUTTY et al. (9)

Series B: d_b = 25mm, C_b = C_s = 2.5cm

Test	ℓ_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
20-12φ5	64.5	660	5230	572	538	1.06
30-12φ5	89.5	470	5360	449	479	0.94
40-12φ5	114.5	370	5280	307	438	0.70
20-9φ6	64.5	770	5310	600	559	1.07
30-9φ6	89.5	560	5400	436	493	0.88
40-9φ6	114.5	430	4920	367	432	0.85
20-5φ8	64.5	650	5690	531	561	0.95
30-5φ8	89.5	470	5280	462	475	0.97
40-5φ8	114.5	370	5150	318	432	0.74
20-4φ10	64.5	840	5500	544	579	0.94
30-4φ10	89.5	600	5700	503	513	0.98
40-4φ10	114.5	470	5760	324	473	0.68
20-12φ5c	64.5	680	2290	345	358	0.96
30-12φ5c	89.5	470	2130	318	301	1.05
40-12φ5c	114.5	390	2160	307	282	1.09
20-12φ6	64.5	910	2790	523	420	1.24
30-12φ6	89.5	740	3390	415	412	1.01
40-12φ6	114.5	510	3700	363	383	0.95
20-6φ10	64.5	1530	4860	613	637	0.96
30-6φ10	89.5	1090	4850	432	542	0.80
40-6φ10	114.5	800	4720	367	472	0.78

TABLE 14. COMPARISON OF CALCULATED AVERAGE BOND STRESSES WITH TEST VALUES: EFFECT OF TRANSVERSE REINFORCEMENT, ROBINSON, ZSUTTY et al. (9)

Series A: d_b = 25mm; C_b = C_s ~ 2.5cm

Test	ℓ_s cm	$\frac{A_{tr}f_{yt}}{sd_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
20-12φ5b	60	720	2220	578	363	1.59
30-12φ5b	85	520	2260	460	319	1.44
40-12φ5b	110	400	2360	380	298	1.27
20-12φ5c	60	650	1270	296	270	1.09
30-12φ5c	85	470	1240	298	232	1.28
20-9φ5	60	550	4620	667	502	1.33
20-9φ5b	60	550	2530	459	372	1.23
30-9φ5b	85	390	2490	418	322	1.30
40-9φ5b	110	300	2430	380	292	1.30
20-6φ5	60	390	4740	578	486	1.19
30-6φ5	85	270	4740	491	428	1.15
20-12φ6	60	960	2860	600	438	1.37
20-9φ8	60	1450	2700	585	477	1.23
30-9φ6	85	509	2990	439	366	1.20
20-7φ6	60	730	4030	681	491	1.39
20-7φ10	60	1745	2230	563	438	1.28
30-5φ10b	85	800	2290	403	348	1.16
30-7φ8	85	820	2330	413	345	1.20
20-5φ8b	60	825	3040	459	426	1.08
30-5φ8b	85	580	2660	397	352	1.13
20-5φ10	60	1120	2000	444	380	1.16
20-5φ10	85	800	2200	471	341	1.38
20-8TT8	60	1940	1950	533	410	1.30
20-10TT6	60	1450	2280	518	438	1.18
30-7TT6	85	720	2350	418	344	1.21
20-7TT6b	60	970	4860	696	573	1.21
20-4TT10b	60	1410	5050	696	647	1.08
20-6TT10	60	2070	2430	541	457	1.18
30-5TT10	85	1220	2360	450	394	1.14
30-6TT8	85	1080	2300	439	376	1.17
20-12TT10	60	4600	2110	696	426	1.63
20-12TT10b	60	4640	1370	541	343	1.51
30-12TT10b	85	3280	1380	492	322	1.53
40-12TT10b	110	2530	1420	380	314	1.21
20-4TT10	60	1630	2190	444	434	1.02
30-4TT10	85	1110	2220	403	372	1.08
40-4TT10	110	850	2280	356	335	1.06
20-4TT10b	60	1410	5050	696	647	1.08

TABLE 15. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: EFFECT OF TRANSVERSE REINFORCEMENT, ROBINSON, ZSUTTY et al. (9)

Test	ℓ_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
Series R: d_b = 20mm; $C_b = C_s$ = 2.0cm						
20-9ϕ5	50	820	2120	405	361	1.12
30-9ϕ5	70	560	2080	365	308	1.19
40-9ϕ5	90	450	2180	348	290	1.20
20-9ϕ5b	50	860	4860	590	552	1.07
20-6ϕ5	50	550	4940	520	512	1.01
30-6ϕ5	70	390	4640	436	436	1.00
30-3ϕ5	70	200	4860	360	419	0.86
40-3ϕ5	90	150	4990	312	396	0.79
20-4TT10	50	2300	1780	356	388	0.92
30-4TT10	70	1640	1880	305	374	0.82
40-4TT10	90	1280	1880	272	341	0.80
20-4TT10b	50	2210	4120	640	590	1.08
30-3TT10	70	1180	4200	422	518	0.82
Series S: d_b = 20mm; $C_b = C_s$ = 2.75cm						
20-9ϕ5	50	830	2500	498	449	1.11
30-9ϕ5	70	560	2580	432	399	1.08
20-4TT10	50	2480	2490	476	515	0.92
30-4TT10	70	1770	2560	427	493	0.87
40-4TT10	90	1375	2630	356	471	0.76
20-4TT10b	50	2275	4690	597	707	0.85
30-3TT10	70	1220	4850	437	640	0.68

TABLE 16. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: EFFECT OF TRANSVERSE REINFORCEMENT, ROBINSON, ZSUTTY et al. (9)

Test	ℓ_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
Series V:	d_b = 25mm; C_b = 4.75; C_s = 3.75cm					
20-6φ5	60	350	3240	518	483	1.07
30-6φ5	85	275	3160	471	434	1.08
40-4φ5	110	130	2990	371	388	0.96
20-4φ8	60	640	5830	674	528	1.28
20-9φ6	60	880	2460	519	473	1.10
30-7TT6	85	720	2400	471	421	1.12
30-7TT10	85	1810	2090	492	465	1.06
40-6φ5	110	210	2320	364	349	1.04
20-10φ10	60	1940	1350	370	396	0.93
30-8φ10	85	1110	1380	329	348	0.94
40-6TT10	110	1280	1390	307	349	0.88
20-12TT8	60	2790	2300	593	517	1.15
30-12TT8	85	2060	2320	492	489	1.01
20-12TT8b	60	2790	1290	370	387	0.96
30-12TT8b	85	1970	1270	314	362	0.87
40-8TT8b	110	1005	1290	275	317	0.87
20-5TT10	60	2000	2050	445	488	0.91
30-8φ10b	85	1350	2050	471	446	1.05
40-5φ8	110	390	2100	364	349	1.04
Series W:	d_b = 25mm; C_b = 2.3cm; C_s = 1.2cm					
20-4φ10b	54.9	930	5060	610	482	1.27
20-4φ10c	54.9	910	2330	385	325	1.18
20-5TT8	54.9	1200	2350	438	354	1.23
20-6φ5c	54.9	470	4040	485	372	1.30
20-5TT10	54.9	2240	2330	405	382	1.06
20-4φ8b	54.9	690	4150	445	304	1.46
20-3TT10	54.9	1350	4710	485	522	0.93
20-12φ6	54.9	1110	2220	484	336	1.44
20-9φ8	54.9	1590	2200	485	371	1.31
20-7TT6	54.9	1100	2320	324	343	0.95
20-12TT10b	54.9	4810	1380	567	294	1.93
30-5φ6c	79.9	320	2400	269	237	1.13
30-5φ8d	79.9	610	3390	386	315	1.22
30-7TT6	79.9	810	2190	334	273	1.23
30-5TT6	79.9	580	2320	278	258	1.08
30-4TT8b	79.9	710	2570	278	285	0.97
30-3φ5	79.9	150	3910	323	281	1.15
30-3φ8	79.9	370	4350	320	326	0.98

TABLE 16 (Continued)

Test	ℓ_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
Series W: d_b = 25mm; C_b = 2.3cm; C_s = 1.2cm						
30-3TT6	79.9	340	4620	334	331	1.01
30-4TT10	79.9	1150	2670	334	336	0.99
30-3TT10	79.9	930	3780	334	372	0.90
30-6ω5d	79.9	320	3270	278	277	1.00
30-4ω10c	79.9	640	2460	278	272	1.02
30-7ω8	79.9	260	2620	389	302	1.29
40-3TT6	104.9	260	4320	297	286	1.04
40-3ω6	104.9	150	1350	297	151	1.96
40-2TT8	104.9	290	4050	249	281	0.89
40-12ω5c	104.9	480	2390	249	234	1.07

TABLE 17. COMPARISON OF CALCULATED AVERAGE BOND STRESS WITH TEST VALUES: EFFECT OF TRANSVERSE REINFORCEMENT, C.U.R. (11)

Test	ℓ_d cm	d_b mm	C_b cm	C_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$	$\frac{u_{top}}{u_{bottom}}$
Series I	14	10	2.3	2.3	3050	2820	670*	780	0.86	0.81
	14	10	2.4	2.4	3050	2820	832	795	1.05	
	14	10	3.7	3.7	3050	2820	920*	811	1.13	
	14	10	5.3	5.3	3050	2820	1110*	811	1.37	1.01
	14	10	5.4	5.4	3050	2820	1100	811	1.37	
	26.5	18	3.2	3.2	1410	2820	548*	680	0.81	0.77
	26.5	18	3.2	3.2	1410	2820	711	672	1.06	
	26.5	18	6.3	6.3	1410	2820	639*	792	0.81	0.73
	26.5	18	6.1	6.1	1410	2820	875	792	1.10	
	26.5	18	9.0	9.0	1410	2820	976*	792	1.23	0.97
	26.5	18	9.0	9.0	1410	2820	1011	792	1.28	
	35	26	2.5	2.5	1060	2820	336*	525	0.64	0.61
	35	26	2.5	2.5	1060	2820	552	525	1.05	
	35	26	6.5	6.5	1060	2820	579*	771	0.75	0.72
	35	26	6.5	6.5	1060	2820	804	771	1.04	
	35	26	10.4	10.4	1060	2820	696*	771	0.90	0.85
	35	26	10.1	10.1	1060	2820	819	771	1.06	
Series II	14	10	1.5	1.5	3050	2840	549*	653	0.84	0.76
	14	10	1.5	1.5	3050	2840	727	653	1.11	
	14	10	1.5	1.5	3050	3510	667*	727	0.92	0.77
	14	10	1.5	1.5	3050	3510	862	727	1.18	
	14	10	1.5	1.5	3050	3680	616*	744	0.83	0.77
	14	10	1.5	1.5	3050	3680	797	744	1.07	
	14	10	1.5	1.5	3050	4960	897*	864	1.04	1.05
	14	10	1.5	1.5	3050	4960	852	864	0.99	
	21	10	1.5	1.5	3050	2570	414*	561	0.74	0.60
	21	10	1.5	1.5	3050	2570	686	561	1.22	
	21	10	1.5	1.5	3050	3820	744*	684	1.09	0.85
	21	10	1.5	1.5	3050	3820	875	684	1.28	
	21	10	1.5	1.5	3050	4040	513*	704	0.73	0.69
	21	10	1.5	1.5	3050	4040	785	704	1.11	
	21	10	1.5	1.5	3050	4960	927*	780	1.19	0.99
	21	10	1.5	1.5	3050	4960	934	780	1.20	
	28	10	1.5	1.5	3050	2480	506*	522	0.97	0.83
	28	10	1.5	1.5	3050	2480	613	522	1.17	

*Top cast bars

TABLE 17 (Continued)

Test	ℓ_d cm	d_b mm	C_b cm	C_s cm	$\frac{A_{tr} f_{yt}}{s d_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$	$\frac{u_{top}}{u_{bottom}}$
Series II (Continued)										
	28	10	1.5	1.5	3050	4520	755*	704	1.07	0.97
	28	10	1.5	1.5	3050	4520	777	704	1.10	
	28	10	1.5	1.5	3050	4600	569*	711	0.80	0.80
	28	10	1.5	1.5	3050	4600	715	711	1.00	
	28	10	1.5	1.5	3050	4910	690*	734	0.94	0.87
	28	10	1.5	1.5	3050	4910	789	734	1.07	
	35	10	1.5	1.5	3050	2930	478*	548	0.87	0.82
	35	10	1.5	1.5	3050	2930	580	548	1.06	
	35	10	1.5	1.5	3050	3070	553*	561	0.99	0.88
	35	10	1.5	1.5	3050	3070	626	561	1.12	

*Top cast bars

TABLE 18. EFFECT OF TOP CASTING ON STRENGTH

Test	ℓ_s in.	d_b in.	C_b in.	C_s in.	$\frac{A_{tr}f_{yt}}{sd_b}$ psi	f'_c psi	u_t psi	u_{cal} psi	$\frac{u_t}{u_{cal}}$
Ferguson and Thompson (12, 13)									
C39	49.4	1.41	2.0	10.06	---	3670	337	416	0.81*
C30	50.75	1.41	4.5	11.62	---	3530	542	599	0.90
C32	50.75	1.41	4.5	11.50	---	3670	499	616	0.82
C37	50.75	1.41	2.0	10.06	---	3040	306	362	0.84*
C36E	33.8	1.41	1.5	3.38	811	3230	416	460	0.90
C28M	33.8	1.41	4.5	5.42	816	3500	610	670	0.91
C29E	33.8	1.41	4.5	5.38	810	3750	626	721	0.87
C24M	50.75	1.41	1.56	5.38	810	2780	350	396	0.88
C31E	67.5	1.41	1.5	5.42	521	3290	335	372	0.90
C34E	67.5	1.41	3.0	5.38	517	3390	434	563	0.77
Thompson, et al. (10)									
8.24.4/2/2.6/6	24	1.0	2.0	2.0	---	2640	497	476	1.04
11.30.4/2/2.6/6	30	1.41	2.0	2.0	---	2910	392	421	0.93

$*C_s/C_bd_b > 3$